

# Impacts of Wetting Layer and Excited State on the Modulation Response of Quantum-Dot Lasers

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**Abstract**—The modulation response of quantum-dot (QD) lasers is studied. Based on a set of four rate equations, a new analytical modulation transfer function is developed via a small-signal analysis. The transfer function can clearly describe the impacts of the wetting layer and the excited states: finite carrier capture and carrier relaxation times as well as the Pauli blocking limits the modulation bandwidth. The definitions of the resonance frequency and the damping factor of QD lasers are also improved. From the analysis, it is demonstrated that carrier escape from the ground state to the excited states leads to a nonzero resonance frequency at low bias powers associated to a strong damping factor.

**Index Terms**—Modulation response, semiconductor laser, quantum-dot (QD).

## I. INTRODUCTION

QUANTUM-DOT lasers have attracted lots of attention as next-generation laser sources for fiber telecommunication networks, because of the promising properties such as low threshold current [1], temperature insensitivity [2], high bandwidth [3], [4] and low chirp [5], [6]. Particularly, directly modulated lasers (DML) have been expected to play a major role in the next-generation telecommunication links for cooler-less and isolator-free applications. However, one of the major drawbacks of QD lasers concern the modulation bandwidth, which remains still limited at room temperature to about 10–12 GHz within 1.3–1.6  $\mu\text{m}$  operating wavelengths [7]. In order to enhance the modulation properties several solutions have been explored including injection-locking [8], tunnelling injection [9] or p-doping [10]. However, for standard QD lasers (namely without any artificial solutions), the modulation bandwidth is still much lower than the best values reported on quantum well (QW) lasers [11]. Consequently, it is essential to clarify the origin of such a limitation. It is well known that the modulation bandwidth is strongly dependent on the resonance frequency as well as on the damping factor.

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The resonance frequency is limited by the maximum modal gain and by gain compression effects [12] while the damping factor much stronger in QD lasers sets the limit of the modulation bandwidth [13]–[15]. Although the underlying physical origins are still under research, gain compression [14], Coulomb interaction [15] and carrier capture dynamics [16]–[18] have been proposed to explain the strong damping in QD lasers. For instance, Asryan et al. also showed that the carrier capture from the optical confinement layer into QDs can strongly limit the modulation bandwidth [19]. It is well-known that the analysis of a semiconductor laser dynamics can be conducted through the analytical expression of the transfer function coming out from the standard two rate equations [12]. However this approach, which is more appropriate to QW lasers is not always suitable for QD devices because of a different and a more sophisticated carrier dynamics. Over the last years, many theoretical studies have been devoted to investigate the carrier dynamics in QD lasers [11], [20]–[23], [35]. On the first hand, one approach relies on the use of pure numerical simulations with coupled differential rate equations [21]–[23]. Although such a method can be very powerful, it does not always give a lot of flexibility to identify the roles of the key-parameters contributing to the laser's degradation properties. On the other hand, Sugawara et. al proposed an empirical expression to fit the experimental modulation response [20] while Fiore et. al developed an indirect approach to recast the set of complex QD rate equations into the standard QW rate equations [11].

In this paper, we theoretically investigate the intensity modulation (IM) properties of InAs/InP(311B) QD lasers [24] with a cascade model including a direct relaxation channel [25]. A new analytical modulation transfer function is derived through a small-signal analysis of the differential rate equations. The analysis gives a good understanding of the impacts of the wetting layer (WL) and of the excited states (ES). It is demonstrated that finite carrier capture time, finite carrier relaxation and Pauli blocking limits the maximum bandwidth. The definitions of the resonance frequency and the damping factor are also improved. The analytical derivation points out that carrier escape from the ground state (GS) to the ES leads to a non-zero resonance frequency at low bias powers and to a strong damping factor as commonly observed in QD lasers. These theoretical investigations are of prime importance for the optimization of low cost sources for optical telecommunications as well as for a further improvement of QD laser performances. The paper is organized as follows. In section II, the numerical model describing QD carrier

dynamics is presented. Then we derive the new modulation transfer function through a small-signal analysis of the differential rate equations. New relationships for the resonance frequency and the damping factor are also demonstrated. In section III, numerical results are presented and compared to experimental ones. Calculations are also compared to those obtained from the standard QW model. Finally, we summarize our results and conclusions in section IV.

## II. NUMERICAL MODEL DESCRIPTION

The numerical model of the QD laser holds under the assumption that the active region consists of only one QD ensemble, where QDs are interconnected by a wetting layer (WL) [25]. The QD ensemble includes two energy levels: a two-fold degenerate ground state (GS) and a four-fold degenerate excited state (ES). The QDs are assumed to be always neutral, electrons and holes are treated as electron-hole (eh) pairs, which mean that the system is in excitonic energy states. Carriers are supposed to be injected directly from the contacts into the WL levels, so the carrier dynamics in the barrier is not taken into account in the model.

Fig. 1 shows the schematic of the carrier dynamics in the conduction band. Firstly, the external injected carrier fills directly into the WL reservoir, some of the carriers are then either captured into the ES within time  $\tau_{ES}^{WL}$  or directly into the GS within time  $\tau_{GS}^{WL}$ , and some of them recombine spontaneously with a spontaneous emission time  $\tau_{WL}^{spon}$ . Once in the ES, carriers can relax into the GS within time  $\tau_{GS}^{ES}$  or recombine spontaneously. On the other hand, carrier can also be thermally reemitted from the ES to the WL with an escape time  $\tau_{WL}^{ES}$ , which is governed by the Fermi distribution for the quasi-thermal equilibrium without external excitation [26]. Similar dynamic behavior is followed for the carrier population on the GS level with regards to the ES. Stimulated emission occurs from the GS when the threshold is reached, and that from the ES is not taken into account in the model. Following the sketch of Fig. 1, the four rate equations on carrier and photon densities are as follows:

$$\frac{dN_{WL}}{dt} = \frac{I}{qV_W} + \frac{N_{ES}}{\tau_{ES}^{ES}} \frac{V_D}{V_W} - \frac{N_{WL}}{\tau_{ES}^{WL}} f_{ES} - \frac{N_{WL}}{\tau_{GS}^{WL}} f_{GS} - \frac{N_{WL}}{\tau_{WL}^{spon}} + \frac{N_{GS}}{\tau_{WL}^{GS}} \frac{V_D}{V_W} \quad (1)$$

$$\frac{dN_{ES}}{dt} = \frac{N_{WL}}{\tau_{ES}^{WL}} \frac{V_W}{V_D} f_{ES} + \frac{N_{GS}}{\tau_{ES}^{GS}} f_{ES} - \frac{N_{ES}}{\tau_{WL}^{ES}} - \frac{N_{ES}}{\tau_{GS}^{ES}} f_{GS} - \frac{N_{ES}}{\tau_{ES}^{spon}} \quad (2)$$

$$\frac{dN_{GS}}{dt} = \frac{N_{WL}}{\tau_{GS}^{WL}} \frac{V_W}{V_D} f_{GS} + \frac{N_{ES}}{\tau_{GS}^{ES}} f_{GS} - \frac{N_{GS}}{\tau_{ES}^{GS}} f_{ES} - \frac{N_{GS}}{\tau_{GS}^{spon}} - g_{GS} v_g S_{GS} - \frac{N_{GS}}{\tau_{WL}^{GS}} \quad (3)$$

$$\frac{dS_{GS}}{dt} = \Gamma_p g_{GS} v_g S_{GS} - \frac{S_{GS}}{\tau_P} + \Gamma_p \beta_{SP} \frac{N_{GS}}{\tau_{GS}^{spon}} \quad (4)$$

where  $N_{WL,ES,GS}$  are carrier densities in WL, ES, GS, and  $S_{GS}$  is photon density in the cavity with GS resonance

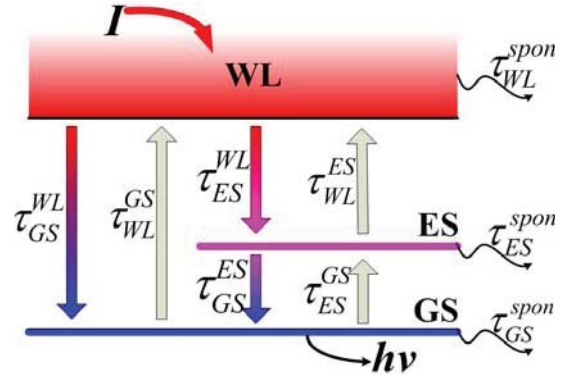


Fig. 1. Sketch of the carrier dynamics model, including a direct relaxation channel.

energy.  $\beta_{SP}$  is the spontaneous emission factor,  $\Gamma_p$  the optical confinement factor,  $\tau_p$  the photon lifetime and  $v_g$  the group velocity.  $V_W$  and  $V_D$  are the volumes of the WL and the QD, respectively. The GS gain is given by:

$$g_{GS} = a_{GS} N_B \left( \frac{N_{GS}}{N_B} - 1 \right) \quad (5)$$

where  $a_{GS}$  is the differential gain and  $N_B$  is the QD density. In what follows, it is important to stress that effects of gain compression are not taken into account. In (1)-(3),  $f_{GS,ES}$  are the Pauli blocking factors of the GS and the ES, respectively, which correspond to the probabilities to find an empty carrier state:

$$f_{GS} = 1 - \frac{N_{GS}}{2N_B}; \quad f_{ES} = 1 - \frac{N_{ES}}{4N_B}. \quad (6)$$

Since the carrier escape from the GS to the WL has little effects on lasing properties [25], the  $N_{GS}/\tau_{WL}^{GS}$  term in (1) and (3) can be neglected.

Based on the rate equations, the corresponding differential rate equations can be derived by considering  $I, N_{WL}, N_{ES}, N_{GS}, S_{GS}$  and  $g_{GS}$  as dynamic variables. In order to simplify the model and to extract the underlying physical mechanism,  $f_{GS,ES}$  are assumed to be constants. To obtain the small-signal responses to a sinusoidal current modulation  $\omega \setminus 2\pi$  the modulation frequency, we assume solutions of the form

$$\begin{aligned} dN_{WL,ES,GS} &= N_{WL,ES,GS} e^{j\omega t} \\ dS_{GS} &= S_{GS} e^{j\omega t}. \end{aligned} \quad (7)$$

Combining (7) into the differential rate equations, we obtain

$$\begin{aligned} &\begin{bmatrix} \gamma_{11} + j\omega & -\gamma_{12} & 0 & 0 \\ -\gamma_{21} & \gamma_{22} + j\omega & -\gamma_{23} & 0 \\ -\gamma_{31} & -\gamma_{32} & \gamma_{33} + j\omega & -\gamma_{34} \\ 0 & 0 & -\gamma_{43} & \gamma_{44} + j\omega \end{bmatrix} \begin{bmatrix} N_{WL1} \\ N_{ES1} \\ N_{GS1} \\ S_{GS1} \end{bmatrix} \\ &= \frac{I_1}{qV_W} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned} \quad (8)$$

with

$$\begin{aligned}
\gamma_{11} &= \frac{f_{ES}}{\tau_{ES}^{WL}} + \frac{f_{GS}}{\tau_{GS}^{WL}} + \frac{1}{\tau_{sp\text{on}}^{WL}}; & \gamma_{12} &= \frac{1}{\tau_{ES}^{WL}} \frac{V_D}{V_W} \\
\gamma_{21} &= \frac{f_{ES}}{\tau_{ES}^{WL}} \frac{V_W}{V_D}; & \gamma_{22} &= \frac{f_{GS}}{\tau_{GS}^{ES}} + \frac{1}{\tau_{WL}^{ES}} + \frac{1}{\tau_{ES}^{sp\text{on}}} \\
\gamma_{23} &= \frac{f_{ES}}{\tau_{GS}^{ES}}; & \gamma_{31} &= \frac{f_{GS}}{\tau_{WL}^{ES}} \frac{V_W}{V_D}; & \gamma_{32} &= \frac{f_{GS}}{\tau_{ES}^{ES}} \\
\gamma_{33} &= \frac{f_{ES}}{\tau_{GS}^{ES}} + \frac{1}{\tau_{sp\text{on}}^{ES}} + v_g a_{GS} S_{GS}; & \gamma_{34} &= -v_g g_{GS} \\
\gamma_{43} &= \frac{\Gamma_p \beta_{SP}}{\tau_{sp\text{on}}^{ES}} + \Gamma_p v_g a_{GS} S_{GS}; & \gamma_{44} &= -\Gamma_p v_g g_{GS} + \frac{1}{\tau_p}.
\end{aligned} \tag{9}$$

Then, we can extract a new modulation transfer function as:

$$H_{QD}(w) = \frac{R_0}{\Delta} \equiv \frac{R_0}{R_0 + jwR_1 - w^2R_2 - jw^3R_3 + w^4} \tag{10}$$

where  $\Delta$  is the determinant of the matrix symbol, and the four parameters which characterize  $H(w)$  are given by:

$$\begin{aligned}
R_0 &= w_R^2 w_{R0}^2 - \gamma_{23} \gamma_{44} (\gamma_{31} \gamma_{12} + \gamma_{11} \gamma_{32}) \\
R_1 &= w_R^2 \Gamma_0 + \Gamma w_{R0}^2 - \gamma_{23} \gamma_{32} (\gamma_{11} + \gamma_{44}) - \gamma_{31} \gamma_{12} \gamma_{23} \\
R_2 &= w_R^2 + \Gamma \Gamma_0 + w_{R0}^2 - \gamma_{23} \gamma_{32} \\
R_3 &= \Gamma + \Gamma_0.
\end{aligned} \tag{11}$$

The relaxation resonance frequency  $w_R$  and damping factor  $\Gamma$  are approximately defined as

$$w_R^2 = \gamma_{33} \gamma_{44} - \gamma_{34} \gamma_{43} \tag{12}$$

$$\Gamma = \gamma_{33} + \gamma_{44}. \tag{13}$$

And the other two new parameters are:

$$w_{R0}^2 = \gamma_{11} \gamma_{22} - \gamma_{12} \gamma_{21} \tag{14}$$

$$\Gamma_0 = \gamma_{11} + \gamma_{22}. \tag{15}$$

Using the set of (9), equations (12) and (13) can be re-expressed as follows:

$$\begin{aligned}
w_R^2 &= \frac{v_g a_{GS} S_{GS}}{\tau_p} + \frac{\Gamma_p \beta_{SP} N_{GS}}{\tau_{GS}^{sp\text{on}} S_{GS}} \left( \frac{f_{ES}}{\tau_{ES}^{GS}} + \frac{1 - \beta_{SP}}{\tau_{GS}^{sp\text{on}}} \right) \\
&\quad + \frac{\beta_{SP}}{\tau_{GS}^{sp\text{on}} \tau_p}
\end{aligned} \tag{16}$$

$$\Gamma = v_g a_{GS} S_{GS} + \frac{f_{ES}}{\tau_{ES}^{GS}} + \frac{1}{\tau_{GS}^{sp\text{on}}} + \frac{\Gamma_p \beta_{SP} N_{GS}}{\tau_{GS}^{sp\text{on}} S_{GS}} \tag{17}$$

where the steady-state relationship  $1/\tau_p - \Gamma_p v_g g_{GS} = \Gamma_p \beta_{SP} N_{GS} / (\tau_{GS}^{sp\text{on}} S_{GS})$  has been used.

Equations (16) and (17) constitute new relations giving the resonance frequency and the damping factor for QD lasers. These equations differ from those obtained from the conventional model of QW lasers, because  $w_R^2$  and  $\Gamma$  contain the additional term  $f_{ES}/\tau_{ES}^{GS}$ , which denotes the carrier escape from the GS to the ES. Since the first term in  $w_R^2$  dominates over all the other terms, the resonance frequency can be

reduced to  $w_R^2 \approx v_g S_{GS} a_{GS} / \tau_p$ . Employing this simplified definition of  $w_R^2$  the damping factor can be rewritten as:

$$\Gamma = K f_R^2 + \frac{f_{ES}}{\tau_{ES}^{GS}} + \frac{1}{\tau_{GS}^{sp\text{on}}} + \frac{\Gamma_p \beta_{SP} N_{GS}}{\tau_{GS}^{sp\text{on}} S_{GS}} \tag{18}$$

where the so-called K-factor is as follows:

$$K = 4\pi^2 \tau_p. \tag{19}$$

The expression of the K-factor is found to be the same as of the conventional one for QW lasers. However, the damping factor  $\Gamma$  in Eq. (18) also contains the term  $f_{ES}/\tau_{ES}^{GS}$  which is comparable to  $K f_R^2$  even at high powers, so the offset can not be neglected.

In order to identify the influences of the WL and the ES, small contribution terms in (11) can be eliminated, then, these equations are simplified as:

$$\begin{aligned}
R_0 &\approx w_R^2 w_{R0}^2; & R_1 &\approx w_R^2 \Gamma_0 + \Gamma w_{R0}^2 \\
R_2 &\approx w_R^2 + \Gamma \Gamma_0 + w_{R0}^2; & R_3 &= \Gamma + \Gamma_0.
\end{aligned} \tag{20}$$

Based on these expressions, the modulation transfer function (10) can be rewritten as

$$H_{app}(w) \approx \left( \frac{w_R^2}{w_R^2 - w^2 + jw\Gamma} \right) \left( \frac{w_{R0}^2}{w_{R0}^2 - w^2 + jw\Gamma_0} \right). \tag{21}$$

This expression reveals that  $w_{R0}^2$  and  $\Gamma_0$  play the same role in the modulation response as the resonance frequency  $w_R^2$  and the damping factor  $\Gamma$ , respectively. It is important to analyze the effects of  $w_{R0}^2$  and  $\Gamma_0$ , as well as the underlying physical mechanism. The results will be discussed in the following section.

Then, the modulation bandwidth  $f_{3dB}$  can be obtained by solving  $|H_{QD}(w)|^2 = 1/2$

$$2R_0^2 = \left( w^4 - R_2 w^2 + R_0 \right)^2 + \left( R_3 w^3 - R_1 w \right)^2 \tag{22}$$

and the maximum possible bandwidth  $f_{3dB}|_{\text{max}}$  occurs when  $\Gamma^2 = 2w_R^2$  so the value can be extracted by:

$$2w_R^4 w_{R0}^4 = \left[ w^4 + w_R^4 \right] \left[ \left( w^2 - w_{R0}^2 \right)^2 + (w\Gamma_0)^2 \right]. \tag{23}$$

### III. RESULTS AND DISCUSSION

#### A. Numerical Results

All the material and QD laser parameters used in our calculations are summarized in Table I.

The capture time  $\tau_{ES}^{WL}$  and relaxation time  $\tau_{GS}^{ES}$  are fixed from time resolved photoluminescence experiment [27]. The direct carrier capture time from the WL to the GS  $\tau_{GS}^{WL}$  is observed to be larger than the capture time  $\tau_{ES}^{WL}$  in the low excitation regime [28], while under strong excitation  $\tau_{GS}^{WL}$  becomes approximately the same as  $\tau_{ES}^{WL}$  ( $\tau_{GS}^{WL} \approx \tau_{ES}^{WL}$ ) [27]. In the calculation, the direct carrier capture time is set to  $\tau_{GS}^{WL} = 1.5\tau_{ES}^{WL}$ . The differential gain value is  $a_{GS} = 0.5 \times 10^{-14} \text{ cm}^2$ .

In order to validate the model, the steady-state properties of the system are at first studied by numerically solving the four rate equations (1)–(4). The results depicted in Fig. 2 show

TABLE I  
MATERIAL PARAMETERS AND LASER PARAMETERS

Symbols	Simulation parameters	Values
$E_{WL}$	WL energy	0.97 eV
$E_{ES}$	ES energy	0.87 eV
$E_{GS}$	GS energy	0.82 eV
$\tau_{ES}^{WL}$	Capture time from WL to ES	25.1 ps
$\tau_{GS}^{ES}$	Relaxation time from ES to GS	11.6 ps
$\tau_{WL}^{spont}$	Spontaneous time of WL	500 ps
$\tau_{ES}^{spont}$	Spontaneous time of ES	500 ps
$\tau_{GS}^{spont}$	Spontaneous time of GS	1200 ps
$n_r$	Refractive index	3.27
$L$	Active region length	0.11 cm
$W$	Active region width	$3 \times 10^{-4}$ cm
$N$	Number of QD layers	5
$N_D$	QD density	$5 \times 10^{10}$ cm $^{-2}$
$\Gamma_p$	Optical confinement factor	0.06
$\beta_{SP}$	Spontaneous emission factor:	$1 \times 10^{-4}$
$\alpha_i$	Internal modal loss	6 cm $^{-1}$
$R_1 = R_2$	Mirror reflectivity	0.3

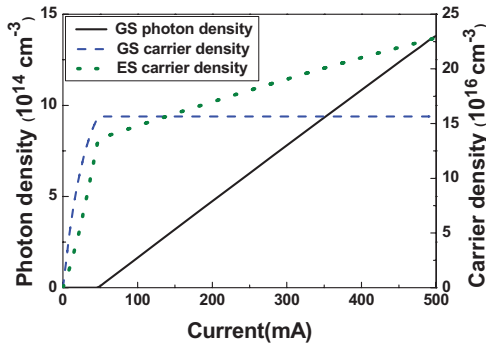


Fig. 2. Photon and carrier densities versus injected current.

that both the GS and the ES carrier populations increase with the injected current. Then, for an injected current larger than 48 mA, the GS population are clamped which leads to the occurrence of the GS lasing emission while the ES population continues to increase with a reduced slope efficiency.

Fig. 3 illustrates the turn-on delay properties for various injected currents. With the increase of the pump current, the delay time becomes shorter which means that the carrier lifetime is decreased [29]. Both the oscillation frequency and the damping factor increase with the current. The oscillation frequencies at  $1.5I_{th}$ ,  $2I_{th}$  and  $4I_{th}$  are 1.94 GHz, 2.64 GHz and 3.52 GHz, respectively.

Based on the steady-state results as well as on the transient response results, the modulation responses are calculated from the analytical expression (10) for various current levels and depicted in Fig. 4. Numerical results show that the relaxation frequency and the damping factor increase with the injected current. At an injected current of around  $2.9I_{th}$ , the modulation bandwidth reaches the maximum value  $\sim 5.5$  GHz, which is in good agreement with the commonly measured values [20]–[32].

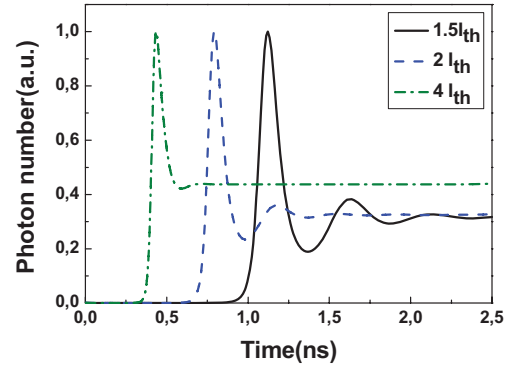


Fig. 3. Turn-on delay properties at various injected currents.

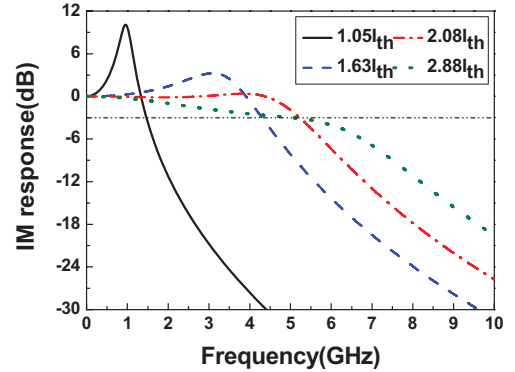


Fig. 4. Modulation responses at several injected currents, which are calculated from the new analytical expression (10).

According to (21), the modulation response  $20 \log |H_{app}(w)|$  can be divided into

$$20 \log |H_{app}(w)| = 20 \log |H_1(w)| + 20 \log |H_0(w)| \quad (24)$$

where

$$H_1(w) = \frac{w_R^2}{w_R^2 - w^2 + jw\Gamma} \quad (25)$$

$$H_0(w) = \frac{w_{R0}^2}{w_{R0}^2 - w^2 + jw\Gamma_0}. \quad (26)$$

The analytical approximation of modulation response at  $2.08I_{th}$  calculated from (24) is shown in Fig. 5 (solid line), and is compared with the exact solution given by (10). Both are found in good agreement except around the resonance peak for which the approximated value is slightly smaller than the exact solution. The behavior of  $H_1(w)$  to the IM response (dash-dot line) is similar to that of QW lasers, and the characteristics of  $w_R^2$  and  $\Gamma$  characterizing  $H_1(w)$  will be discussed in the following section. Since the modulation bandwidth of  $H_0(w)$  is much smaller than that of  $H_1(w)$ , the total modulation bandwidth of the QD laser is limited by  $H_0(w)$ . According to the expression of  $w_{R0}^2$  in  $H_0(w)$  associated with (9), the results point out that the finite carrier capture time  $\tau_{ES}^{WL}$  and  $\tau_{GS}^{WL}$ , finite carrier relaxation time  $\tau_{GS}^{ES}$  as well as Pauli blocking factor  $f_{ES}$  and  $f_{GS}$  are the underlying physical limitations for the enhancement of modulation bandwidth. In order to further analyze the characteristics of  $f_{R0}^2$  and  $\Gamma_0$ , their evolutions are

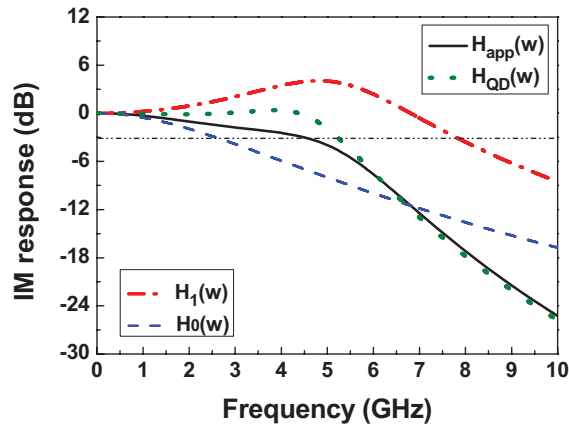


Fig. 5. Comparisons of analytical approximation (solid) calculated from (24) for modulation response and the precise value (dot) calculated from (10). Dashed line indicates contribution of  $H_0(w)$  to the total modulation response, and dashed dotted line is contribution of  $H_1(w)$ .

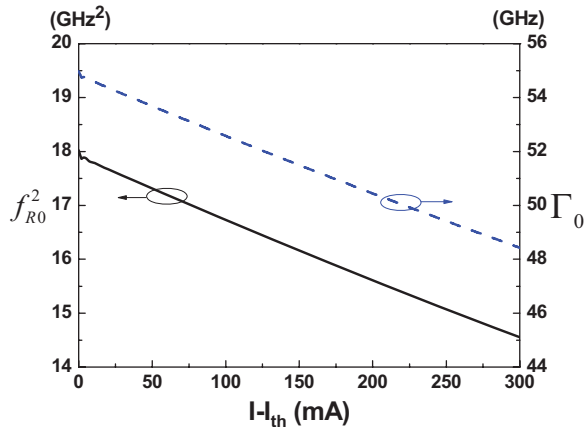


Fig. 6. Evolutions of  $f_{R0}^2$  and  $\Gamma_0$  as a function of the current ( $I-I_{th}$ ).

plotted in Fig. 6 as a function of the current ( $I-I_{th}$ ). Inversely to the behaviors of  $f_R^2$  and  $\Gamma$ , both  $f_{R0}^2$  and  $\Gamma_0$  decrease linearly with the increased current. This is attributed to the reduced Pauli blocking factor  $f_{ES}$ . The relation of the two parameters is fitted as  $\Gamma_0 = 1.9f_{R0}^2 + 20.8$  (GHz), and the maximum  $f_{R0}$  at threshold is only 4.2 GHz associated with a large  $\Gamma_0$  value of 55.0 GHz, which result in the small bandwidth of  $H_0(w)$ .

### B. Comparison to Experimental Results

The device under study is an InAs/InP(311B) QD laser [24], where the heterostructure is grown by molecular beam epitaxy (MBE) on a (311)B oriented InP substrate. The active region consists of 5 QD layers, and the measured QD density is  $\sim 10^{11}$  cm $^{-2}$  [33]. The length and width of the ridge wave-guide laser are 1.1 mm and  $3 \times 10^{-3}$  mm, respectively. The laser's facets are as-cleaved. The experiment shows that the GS lasing peaks at 1.52  $\mu$ m at room temperature under continuous wave (cw) operation. The photon lifetime is measured to be about 5.8 ps. In this section, we use the new analytical transfer function (10) to simulate the laser modulation performance. In the calculations, the differential gain  $a_{GS}$  is the only one

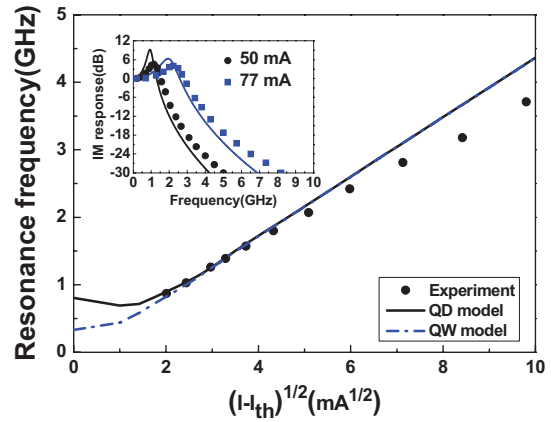


Fig. 7. Resonance frequency as a function of the current ( $I-I_{th}$ ) $^{1/2}$  and intensity modulation response at 50 and 77 mA (inset). Dots denote experimental results. Solid lines are theoretical results from QD model, compared with those from QW model (dashed dotted lines).

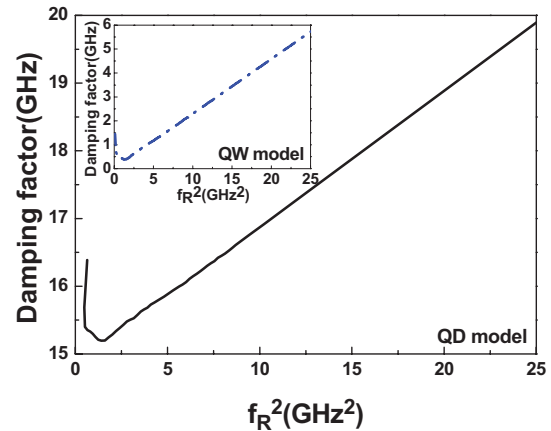


Fig. 8. Comparisons of damping factors calculated from the QD model and the QW model.

fitting parameter, which is adjusted to  $0.25 \times 10^{-14}$  cm $^2$ . All other parameters are set to the experimental values.

Fig. 7 depicts the resonance frequency  $f_R$  as a function of the current  $(I-I_{th})^{1/2}$ , while the inset shows the modulation response at two different pump currents (50 mA and 77 mA). Theoretical results (solid lines) obtained from the QD model lead to a relative good agreement with the experimental ones (dot lines). However, at large current injections, the calculated resonance frequency (lines) is found to be higher than the experimental one. Such a discrepancy is attributed to the gain compression which is not considered in the model. Analytical calculations also point out that the carrier escape from the GS to the ES induces a non-zero resonance frequency around 1 GHz at low bias powers. Based on (16), the larger  $\tau_{GS}^{ES}$  the smaller the frequency offset at threshold. This resonance frequency offset is larger than the one due to spontaneous emission only in QW lasers (dash-dot line). In the inset, the theoretical modulation responses (solid lines) match relatively well with the experimental results (dots).

Fig. 8 shows the evolution of damping factor  $\Gamma$  as a function of the resonance frequency  $f_R^2$ . According to (18), their relationship can be fitted as  $\Gamma = 0.20f_R^2 + 14.9$  (GHz),

in comparison with the result from the conventional QW model (inset) which is  $\Gamma = 0.23f_R^2 + 0.066$  (GHz). The two K-factors are nearly the same  $\sim 0.2$  ns, which is smaller than the experimental value (0.6 ns) [24]. Qualitatively, such a discrepancy can be partly attributed to the fact that the simulation does not take into account the gain saturation effects, which can be comparable to the one related to the differential gain  $a_{GS}$  [12]. Besides, parasitic RC and carrier transport effects as well as the temperature effect also contribute to the discrepancy. The offset occurring in QD model is found to be much larger than that of QW model, confirming the strong damping in QD lasers. Let us stress that such a strong damping has also been pointed out to explain the QD laser's insensitivity to external perturbations [14], [18]. According to (17), carrier escape from the GS to the ES ( $f_{ES}/\tau_{ES}^{GS}$ ) is responsible for this large damping factor. The deviation from linearity at low relaxation resonance frequency is attributed to the spontaneous emission term  $(\Gamma_p \beta_{SP} N_{GS})/(\tau_{GS}^{spont} S_{GS})$  in the damping factor expression. This phenomenon has been observed in InGaAsP bulk lasers via a parasitic-free optical modulation technique [34].

#### IV. CONCLUSION

Based on a set of four rate equations, a new analytical modulation transfer function of QD lasers has been introduced via a small-signal analysis. This numerical study clarifies the roles of the WL and of the ES to the modulation response: finite carrier capture time, finite carrier relaxation time and Pauli blocking have been found to be physical limitations to the enhancement of the modulation bandwidth. The model has been used to recast the the definitions of the resonance frequency and of the damping factor. Calculations show that carrier escape from the GS to the ES gives rise to a non-zero resonance frequency at low bias powers as well as to a strong damping factor. These results are of prime importance for further improvements of QD laser dynamic properties. Further studies will improve the rate equation model such as including the gain compression effect, the ES lasing, and treating Pauli blocking factors as variables in the differential equations.

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