Journal of Applied Physics

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Cite as: J. Appl. Phys. **127**, 073104 (2020); https://doi.org/10.1063/1.5124521
Submitted: 14 August 2019 . Accepted: 05 February 2020 . Published Online: 19 February 2020

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Journal of Applied Physics ARTICLE scitation.org/journal/jap

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Submitted: 14 August 2019 · Accepted: 5 February 2020 ·
Published Online: 19 February 2020

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ABSTRACT

In this work, we propose to employ strong optical feedback to narrow the spectral linewidth of quantum cascade lasers without using any phase control. Rate equation analysis demonstrates that optical feedback beyond a certain level always reduces the laser linewidth for any feedback phase. It is also found that the linewidth becomes less sensitive to the feedback phase for higher feedback strength. Simulations show that optical feedback with a feedback ratio of −10 dB can suppress the laser linewidth by about two orders of magnitude. This is in contrast to near-infrared laser diodes, which can be easily destabilized by strong feedback.

I. INTRODUCTION

Many molecules including carbon oxide and methane have strong absorption lines in the mid-infrared range, owing to the vibrational and rotational transitions.[1] Mid-infrared quantum cascade lasers (QCLs) have become popular for molecule gas sensing since the invention in 1994.[2] The precision and resolution of molecule spectroscopy highly relies on the spectral linewidth of the QCLs.[3] Thanks to the near-zero linewidth broadening factor (LBF),[4] the QCLs own narrow intrinsic linewidth of a few hundred hertz, which is close to the Schawlow–Townes limit.[5] However, the flicker noise ($1/f$ noise) arising from current source noise, temperature fluctuation, and mechanical vibration significantly broadens the practical linewidth of the free-running QCLs up to the megahertz range.[6] In order to narrow the spectral linewidth of QCLs down to kilohertz or hertz range, a large variety of frequency stabilization techniques have been developed. One main method is frequency locking a QCL to one side of a molecular absorption line, whereas the price to pay is the loss of wavelength tunability.[7,8] The other method is locking one QCL to a high-finesse optical cavity through the Pound–Drever–Hall approach, which suffers from external acoustic and mechanical vibrations.[9–11] A more common method is phase locking the QCL to a near-infrared optical frequency comb (OFC) through either difference- or sum-frequency generation process.[12–15] A remarkable linewidth reduction down to the sub-Hz level has been achieved using this method.[15] However, the corresponding experimental setup is very complicated and bulky, which can hinder the practical applications. In addition, there are also some other approaches including phase locking to a narrow-linewidth CO_2 laser,[16,17] optical injection locking to an OFC,[18,19] and pure electrical feedback.[20,21] It is noted that all the above methods require complex active feedback control. In addition, resonant optical feedback offered by a high-finesse optical cavity has been used to narrow the linewidth of QCLs, which is very helpful for high-sensitivity cavity-enhanced absorption spectroscopy applications.[22–24] However, the feedback phase has to be finely controlled through a piezoelectric transducer to meet the linewidth narrowing condition. This work proposes to use strong optical feedback for reducing the spectral linewidth of QCLs, without any feedback phase control. With respect to the complicated optical frequency comb stabilization technique, the proposed method is much more simple, and is very cost-effective.

Optical feedback is usually detrimental to the stability of semiconductor lasers, because feedback with strength above a certain level named *critical feedback level* drives the laser into coherence collapse or chaos, where both phase noise and intensity noise are substantially increased.[25,26] Therefore, commercial laser diodes are often accompanied by an optical isolator to prevent residual optical feedback in optical links. The critical feedback level of bulk and quantum well lasers are in the range of −30 to −40 dB,[27–29] while quantum dot lasers are less sensitive to optical feedback with a critical feedback level up to −7.4 dB.[30] Weak optical feedback below the critical feedback level either broadens or narrows the spectral linewidth depending on the feedback phase.[31–33] Therefore, precise control of the feedback phase is required to reduce the linewidth of semiconductor lasers. In contrast, QCLs are extensively proved to be more robust against optical feedback than normal diode lasers. The enhanced stability of QCLs subject to optical feedback was underlined by the fact that the stable regimes are much broader than in the case of interband diode lasers.[34–37] For instance, the third regime in QCLs is relatively broad, whereas it does not always appear in diode lasers.[37]

Very recently, we experimentally showed that one mid-infrared QCL was stable with feedback ratios up to a feedback level of −5.0 dB. The laser remains single-mode lasing and the feedback-induced relative intensity noise fluctuation is less than 2.0 dB.[38] Such a high feedback tolerance motivates the application of strong optical feedback to reduce the spectral linewidth of QCLs in this article. The spectral linewidth characteristics of QCLs subject to strong optical feedback are investigated through the small-signal analysis of the QCL rate equations with Langevin noise sources.

II. RATE EQUATION MODEL

Figure 1 shows the schematic of the simplified carrier dynamics in one gain stage of QCLs. Carriers from the injector region are first injected into the upper subband. Then, some carriers transit into the lower subband with a scattering time of τ_{32}, while the others transit into the bottom subband with a scattering time of τ_{31}. The stimulated emission relies on the population inversion between the upper and lower subbands. Carriers in the lower subbands scatter into the bottom subband with a lifetime of τ_{21}, and finally leave the gain stage to the subsequent one with a tunneling out time of τ_{out}. Following Fig. 1, the rate equations describing the carrier dynamics in the upper level (N_3), lower level (N_2), and bottom level (N_1), respectively, read[39]

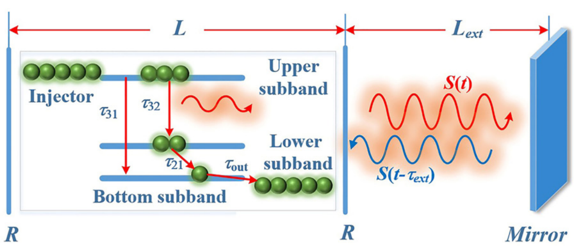

FIG. 1. Schematic of the carrier dynamics in QCLs, and illustration of optical feedback. L is the laser cavity length, R is the facet reflectivity, and L_{ext} is the single-trip feedback length.

$$\frac{dN_3}{dt} = \eta\frac{I}{q} - \frac{N_3}{\tau_{32}} - \frac{N_3}{\tau_{31}} - G_0\Delta N S + F_3(t), \tag{1}$$

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} + G_0\Delta N S + F_2(t), \tag{2}$$

$$\frac{dN_1}{dt} = \frac{N_3}{\tau_{31}} + \frac{N_2}{\tau_{21}} - \frac{N_1}{\tau_{out}} + F_1(t), \tag{3}$$

where I is the pump current, η is the current injection efficiency, G_0 is the gain coefficient, and ΔN is the population inversion defined as $\Delta N = N_3 - N_2$. The electric field of the QCL subject to optical feedback is described by the classical Lang–Kobayashi model,[40] in which the photon number (S) and the phase (ϕ) of the electric field are described by

$$\begin{aligned}\frac{dS}{dt} &= (mG_0\Delta N - 1/\tau_p)S + m\beta\frac{N_3}{\tau_{sp}} + F_S(t)\\ &\quad + 2k_c\sqrt{r_{ext}S(t-\tau_{ext})S(t)}\cos\Delta\phi,\end{aligned} \tag{4}$$

$$\frac{d\phi}{dt} = \frac{\alpha_H}{2}(mG_0\Delta N - 1/\tau_p) + F_\phi(t) - k_c\sqrt{\frac{r_{ext}S(t-\tau_{ext})}{S(t)}}\sin\Delta\phi, \tag{5}$$

where m is the number of gain stages, τ_p is the photon lifetime, β is the spontaneous emission factor, τ_{sp} is the spontaneous emission time, and α_H is the LBF. The feedback coupling coefficient is given by $k_c = c(1-R)/\left(2n_r L\sqrt{R}\right)$, with c being the light velocity, n_r being the refractive index, L being the cavity length, and R being the facet reflectivity. r_{ext} is the feedback ratio of the reflected light power at the front facet to the emitted light power without optical feedback. τ_{ext} is the round-trip delay time with $\tau_{ext} = 2L_{ext}/c$, with L_{ext} being the external cavity length. The phase difference $\Delta\phi$ is described by $\Delta\phi = \phi_0 + \phi(t) - \phi(t-\tau_{ext})$, where ϕ_0 is the initial phase $\phi_0 = \omega_0 \cdot \tau_{ext}$, with ω_0 being the lasing frequency of the free-running QCL. Because the initial phase is extremely sensitive to the feedback delay time, we treat ϕ_0 as a free parameter in this work.[41] The carrier noise $F_{3,2,1}$ and the spontaneous emission noise $F_{S,\phi}$ are described by the Langevin approach.[42–44] It is remarked that the Langevin noise sources are white noise, while the flicker noise is not considered in this work.[45,46] The Langevin noise sources perturb the laser system away from the steady-state solutions. Applying the standard small-signal analysis to rate Eqs. (1)–(5),[43,44] we obtain variations of the carriers (δN_3, δN_2), the photon (δS), and the phase ($\delta\phi$) in the frequency domain

$$\begin{bmatrix} j\omega+\gamma_{11} & -\gamma_{12} & \gamma_{13} & 0 \\ -\gamma_{21} & j\omega+\gamma_{22} & -\gamma_{23} & 0 \\ -\gamma_{31} & \gamma_{32} & j\omega-\gamma_{33} & \gamma_{34} \\ -\gamma_{41} & \gamma_{42} & -\gamma_{43} & j\omega+\gamma_{44} \end{bmatrix} \begin{bmatrix} \delta N_3(\omega) \\ \delta N_2(\omega) \\ \delta S(\omega) \\ \delta\phi(\omega) \end{bmatrix} = \begin{bmatrix} F_3 \\ F_2 \\ F_S \\ F_\phi \end{bmatrix}, \tag{6}$$

with the parameters

$$\begin{aligned}
&\gamma_{11} = \tau_{32}^{-1} + \tau_{31}^{-1} + G_0 S,\ \gamma_{12} = G_0 S,\ \gamma_{13} = G_0 \Delta N,\\
&\gamma_{21} = \tau_{32}^{-1} + G_0 S,\ \gamma_{22} = \tau_{21}^{-1} + G_0 S,\ \gamma_{23} = G_0 \Delta N,\\
&\gamma_{31} = m(\beta \tau_{sp}^{-1} + G_0 S),\ \gamma_{32} = m G_0 S,\\
&\gamma_{33} = m G_0 \Delta N - \tau_p^{-1} + k_c \sqrt{r_{ext}}(1 + e^{-j\omega\tau_{ext}})\cos(\phi_0 + \Delta\omega_s \tau_{ext}),\\
&\gamma_{34} = 2k_c \sqrt{r_{ext}}(1 - e^{-j\omega\tau_{ext}})\sin(\phi_0 + \Delta\omega_s \tau_{ext})S,\\
&\gamma_{41} = \frac{\alpha_H}{2} m G_0,\ \gamma_{42} = \frac{\alpha_H}{2} m G_0,\\
&\gamma_{43} = \frac{k_c \sqrt{r_{ext}}}{2S}(1 - e^{-j\omega\tau_{ext}})\sin(\phi_0 + \Delta\omega_s \tau_{ext}),\\
&\gamma_{44} = k_c \sqrt{r_{ext}}(1 - e^{-j\omega\tau_{ext}})\cos(\phi_0 + \Delta\omega_s \tau_{ext}),
\end{aligned} \tag{7}$$

where $\Delta\omega_s$ is the frequency shift induced by the optical feedback. According to Cramer's rule, the frequency noise power spectral density (FNPSD) of the QCL is calculated by $FN(\omega) = |j\omega\delta\phi/(2\pi)|^2$.[42] Carrier dynamics of the bottom subband is not included in the above linearized rate equations, because N_1 does not contribute to the FNPSD. All the QCL parameters used for the simulations in this work are listed in Table I, unless stated otherwise.[43,44]

III. RESULTS AND DISCUSSION

The QCL under study exhibits a lasing threshold of $I_{th} = 223$ mA. The FNPSD of the free-running QCL exhibits a broad peak around 100 GHz as shown in Fig. 2 due to the contribution of carrier noise.[43] The low-frequency (<1.0 GHz) FN of the free-running laser biased at $1.5 \times I_{th}$ is constant at 933.0 Hz^2/Hz due to the nature of white noise. When the QCL is subject to optical feedback, ripples appear in the FNPSD with a frequency interval determined by the inverse of the feedback delay time ($\tau_{ext} = 1.0$ ns). For the in-phase feedback ($\phi_0 = 0$) in Fig. 2(a), increasing the feedback strength always reduces the low-frequency FN. However, this is not the case for the out-of-phase feedback

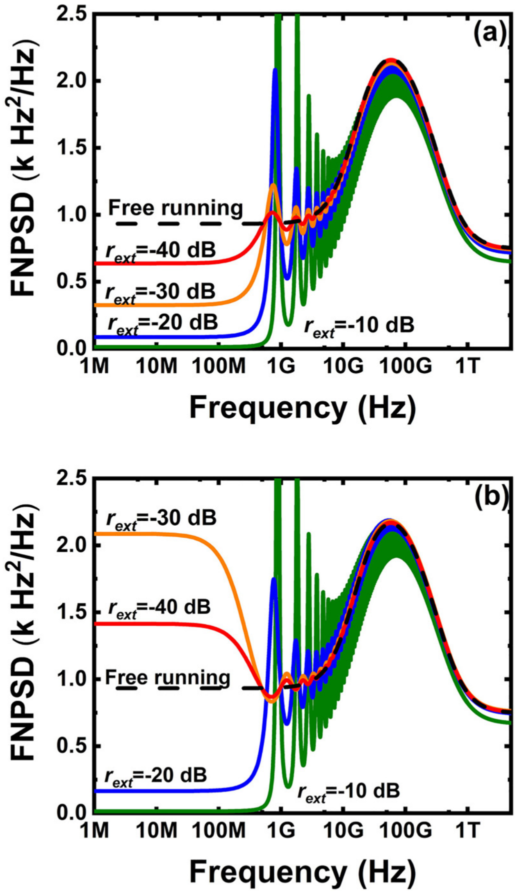

FIG. 2. Simulated FNPSD of the QCL subject to optical feedback with an initial phase of (a) $\phi_0 = 0$ and of (b) $\phi_0 = \pi$. The pump current is set at $1.5 \times I_{th}$.

($\phi_0 = \pi$) in Fig. 2(b), where the low-frequency FN is increased by weak feedback ($r_{ext} = -30$, -40 dB) and is suppressed by strong feedback ($r_{ext} = -20$, -10 dB). Therefore, the initial feedback phase has a significant influence on the FNPSD of QCLs as near-infrared laser diodes.[28,31]

The low-frequency FN in Fig. 2 determines the intrinsic linewidth of QCLs by $\Delta\nu = 2\pi FN(\omega \to 0)$.[26] By analyzing the linearized Eq. (6), we analytically derive the intrinsic linewidth of QCL with optical feedback as

$$\Delta\nu = \frac{m(G_0 S + \beta/\tau_{sp})N_3}{4\pi S^2} \frac{1 + \alpha_H^2}{[1 + C\cos(\phi_0 + \Delta\omega_s \tau_{ext} + \tan^{-1}\alpha_H)]^2}, \tag{8}$$

where the C parameter and the feedback-induced frequency shift $\Delta\omega_s$ are given by

$$\begin{aligned}
C &= k_c \tau_{ext} \sqrt{r_{ext}(1 + \alpha_H^2)},\\
\Delta\omega_s &= -k_c \sqrt{r_{ext}(1 + \alpha_H^2)}\sin(\phi_0 + \tan^{-1}\alpha_H + \Delta\omega_s \tau_{ext}).
\end{aligned} \tag{9}$$

TABLE I. QCL parameters used in the simulations.

Symbol	Description	Value
G_0	Gain coefficient	5.3×10^4 /s
τ_p	Photon lifetime	3.7 ps
τ_{sp}	Spontaneous emission time	7.0 ns
β	Spontaneous emission factor	1.0×10^{-6}
α_H	Linewidth broadening factor	0.5
m	Gain stage number	30
τ_{32}	Scattering time upper to lower	2.0 ps
τ_{31}	Scattering time upper to bottom	2.4 ps
τ_{21}	Scattering time lower to bottom	0.5 ps
τ_{out}	Tunneling out time	0.5 ps
L	Cavity length	3.0 mm
R	Facet reflectivity	0.29
n_r	Refractivity index	3.3
L_{ext}	External cavity length	15 cm

In Eq. (8), the term β/τ_{sp} is negligible because it is much smaller than the term G_0S. The relation of the feedback linewidth $\Delta\nu$ with the free-running linewidth $\Delta\nu_{free}$ is

$$\Delta\nu \approx \frac{A}{[1 + C\cos(\phi_0 + \Delta\omega_s\tau_{ext} + \tan^{-1}\alpha_H)]^2}\Delta\nu_{free}, \tag{10}$$

with

$$\begin{aligned} A &= (1-\kappa)\left[1 - \frac{\kappa B}{B-\kappa}\left(\frac{B-1}{I/I_{th}-1+B} + \frac{1-\kappa}{I/I_{th}-1+\kappa}\right)\right],\\ B &= \frac{\tau_{32}(\tau_{31}+\tau_{21})}{\tau_{21}(\tau_{31}+\tau_{32})}, \\ \kappa &= 2k_c\tau_p\sqrt{r_{ext}}\cos(\phi_0 + \Delta\omega_s\tau_{ext}), \end{aligned} \tag{11}$$

where the influence of optical feedback on the Langevin noise sources has been taken into account in Eq. (10). In case the perturbation of optical feedback on the Langevin noise sources is not considered, this formula is reduced to the conventional one as that of near-infrared laser diodes,[33,47]

$$\Delta\nu_c \approx \frac{\Delta\nu_{free}}{[1 + C\cos(\phi_0 + \Delta\omega_s\tau_{ext} + \tan^{-1}\alpha_H)]^2}. \tag{12}$$

For a certain group of feedback ratios and feedback delays, the minimum linewidth is achieved at $\phi_0 = -\tan^{-1}(\alpha_H)$, while the maximum linewidth is obtained at $\phi_0 = \pi - \tan^{-1}(\alpha_H)$. Meanwhile, the frequency shift $\Delta\omega_s$ is zero for both initial phases.

The simulated QCL shown in Fig. 3 remains stable for feedback ratios up to −10 dB, which suggests that the critical feedback level of the QCL is above −10 dB. The map in Fig. 3(a) illustrates the feedback-induced linewidth reduction, which is defined as the ratio of the linewidth of free-running QCL to the linewidth of the laser with optical feedback. For weak optical feedback with $C < 1$ ($r_{ext} < -27.0$ dB), the linewidth is highly sensitive to the initial feedback phase, and can be either broadened or narrowed depending on the phase value. This is similar to the case of near-infrared laser diodes subject to optical feedback.[26] However, the QCL linewidth becomes much less sensitive to the initial phase for strong optical feedback with $C > 1$ ($r_{ext} > -27.0$ dB). In addition, strong optical feedback always reduces the intrinsic linewidth for any feedback phase. The higher the feedback ratio, the linewidth is less sensitive to the feedback phase. At a feedback ratio of −10 dB, the linewidth is almost independent on the feedback phase, and the linewidth reduction reaches as high as about 18.8 dB. This feature enables the usage of strong optical feedback to reduce the linewidth of QCLs without any phase control. It is reminded that the possibility of using strong feedback is benefited from the high stability of QCLs against optical feedback.[38] The maximum linewidth of the QCL is reached at the intersection of $C = 1$ (dotted line) and $\phi_0 = \pi - \tan^{-1}(\alpha_H)$ (dashed-dotted line). Figure 3(b) shows linewidth reduction as a function of the normalized pump current, with a feedback ratio of $r_{ext} = -10$ dB. It is proved that the linewidth reduction obtained from the analytical formula Eq. (10) (star) quantitatively matches well with that calculated from Eq. (6) (dots). We found that optical feedback is more efficient to narrow the linewidth of the QCL operated near threshold. The linewidth reduction goes down from 21.0 dB at $1.05 \times I_{th}$ to 18.4 dB at $4.0 \times I_{th}$. This is similar to the feedback effect on the relative intensity noise, which gains the largest reduction near the threshold.[48] In comparison, the linewidth reduction obtained from the conventional formula Eq. (12) (triangle) is almost constant around 18.0 dB for all the pump currents.

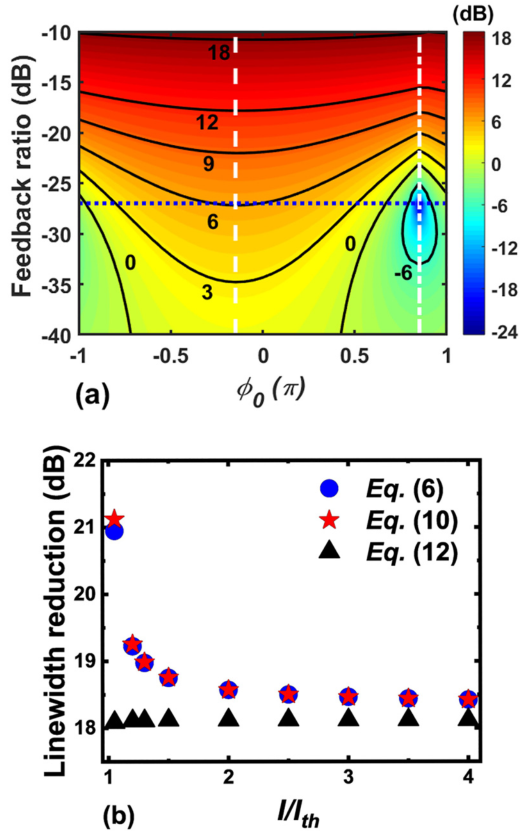

FIG. 3. (a) Linewidth reduction map as functions of feedback ratio and initial phase. The pump current is set at $1.5 \times I_{th}$. The dashed line indicates $\phi_0 = -\tan^{-1}(\alpha_H)$, the dashed-dotted line indicates $\phi_0 = \pi\text{-}\tan^{-1}(\alpha_H)$, and the dotted line indicates $C = 1$. (b) The linewidth reduction vs the pump current at $r_{ext} = -10$ dB.

Figure 4 shows that longer external cavity length induces higher ripples at lower Fourier frequencies of the FNPSD. On the other hand, longer length results in larger linewidth reduction as shown in the inset of Fig. 4. For a feedback ratio of $r_{ext} = -10$ dB, the linewidth reduction increases from 5.2 dB at $L_{ext} = 1.5$ cm up to 18.8 dB at $L_{ext} = 15.0$ cm. Therefore, longer external cavity length is desirable to reduce the linewidth of QCLs. It is remarked that the ripples in the FNPSD result in the appearance of satellite peaks in the corresponding optical spectrum, which act as side modes. However, the amplitude of

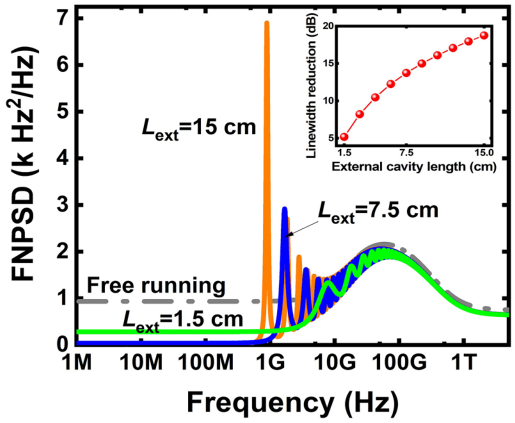

FIG. 4. In-phase optical feedback effects on the QCL FNPSD for several external cavity lengths with $r_{ext} = -10$ dB. Inset is the linewidth reduction vs the external cavity length. The pump current is set at $1.5 \times I_{th}$.

the side modes is much lower than the main mode (83 dB lower for the case of $L_{ext} = 15.0$ cm), which hardly affects practical applications.[44]

We did not observe any pulse instability with optical feedback, for the QCL pumped at $1.5 \times I_{th}$. In literature studies, it has been pointed out that a QCL operated close to the lasing threshold associated with a large LBF was more easily destabilized by the optical feedback.[26,36] Experiments have shown that the measured LBFs of QCLs above the threshold range from 0 to 3.0.[49,50] Therefore, we studied the feedback effects on the linewidth of the QCL pumped at $1.05 \times I_{th}$ with $\alpha_H = 0.5$ and $\alpha_H = 2.0$, respectively, as shown in Fig. 5. The maps in Fig. 5 illustrate that the longer external cavity length leads to larger linewidth reduction for both in-phase and out-of-phase feedbacks. This is because longer feedback delay enhances the feedback C parameter in Eq. (9) and hence reduces the linewidth expressed in Eqs. (10) and (12). Physically speaking, longer feedback length raises the effective photon lifetime of the coupled cavity, and thus narrows the QCL's linewidth owing to the higher quality factor of the cavity. For the same in-phase feedback conditions, the linewidth reduction for a high LBF of $\alpha_H = 2.0$ in Fig. 5(c) is larger than that for a low one of $\alpha_H = 0.5$ in Fig. 5(a). Interestingly, a small regime of destabilized period-one oscillation with a frequency of 0.93 GHz appears in the vicinity of $r_{ext} = -10$ dB and $L_{ext} = 15.0$ cm (blank region) in Fig. 5(c), which suggests that the oscillation frequency is close to the external cavity frequency. On the other hand, out-of-phase feedback with $\alpha_H = 2.0$ in Fig. 5(d) leads to a wider region of periodic and quasiperiodic oscillations (blank region). Therefore, a larger LBF associated with a longer external cavity length is more likely to destabilize the QCL operated close to the lasing threshold.[36] For all the above conditions, the QCL linewidth is always reduced by optical feedback when the feedback parameter $C > 1$, as that shown in Fig. 3.

Finally, it is remarked that although the rate equation model in this work only considers the white noise, the strong optical feedback suppression effect on the FN is extendable to flicker noise as well.[26] For the experimental implementation, one can employ a reflection mirror to provide the strong optical feedback, and a beam splitter (placed between the QCL and the mirror) to direct the laser output.[38] The corresponding experimental demonstration will be reported elsewhere.

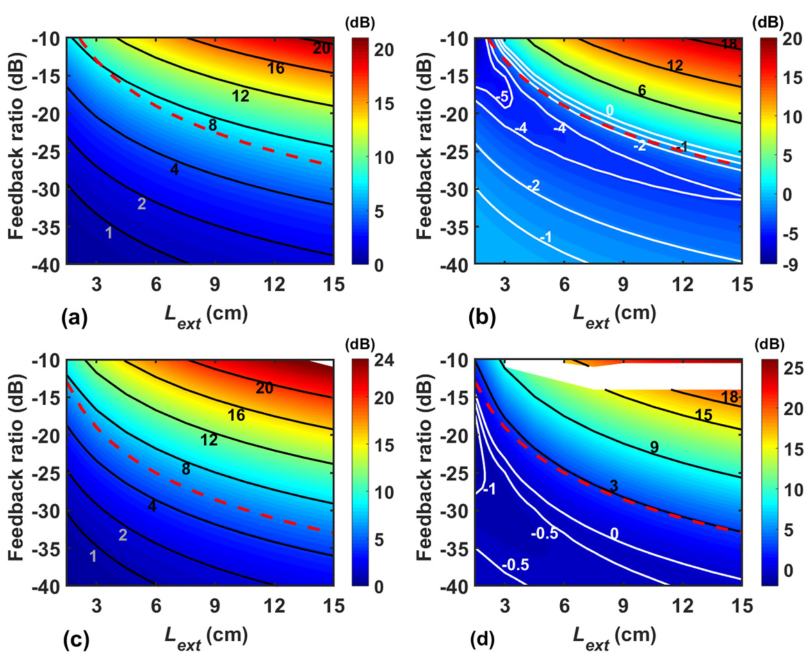

FIG. 5. Linewidth reduction map with respect to external cavity length and feedback ratio for (a) $\alpha_H = 0.5$, $\phi_0 = 0$; (b) $\alpha_H = 0.5$, $\phi_0 = \pi$; (c) $\alpha_H = 2.0$, $\phi_0 = 0$; and (d) $\alpha_H = 2.0$, $\phi_0 = \pi$. The dashed line indicates $C = 1$. The pump current is fixed at $1.05 \times I_{th}$.

IV. CONCLUSION

In summary, we theoretically investigated the FN and the linewidth characteristics of QCLs subject to optical feedback through a small-signal analysis of the rate equations. It is found that strong optical feedback with the *C* parameter beyond one always narrows the linewidth of QCLs for any feedback phase condition. The linewidth value becomes less dependent on the feedback phase for stronger feedback strength. Optical feedback with a feedback ratio of −10 dB can narrow the linewidth by about 20 dB. In addition, a long feedback length is desirable to further reduce the spectral linewidth. We also demonstrate that a QCL operated close to the lasing threshold is more likely to be destabilized by a large LBF in combination with a long external feedback length, which produces both periodic and aperiodic oscillations.

ACKNOWLEDGMENTS

This work was supported by the Shanghai Pujiang Program (No. 17PJ1406500) and National Natural Science Foundation of China (61804095).

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