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Template Attacks, Optimal Distinguishers & Perceived Information Metric

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Overview

Outlines

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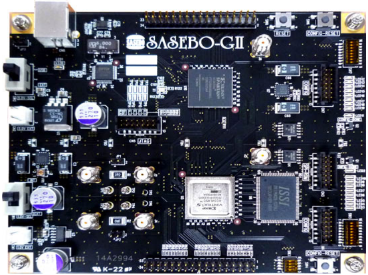
- Use the *maximum likelihood* (ML) to derive MIA and the (experimental) template attack in case of profiling
- Experiments: should theoretical values of probabilities be used or should they be estimated on-the-fly?

Motivation

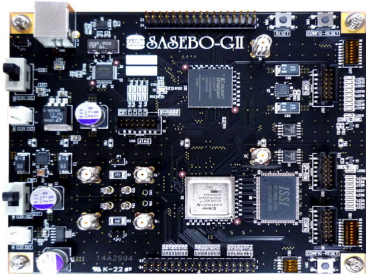
Profiling device



$\hat{\mathbb{P}}$ for an estimation offline

$\rightarrow \mathbb{P}$ exact probability

Attacking device



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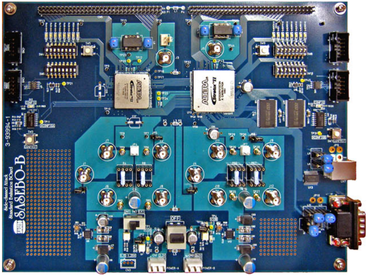	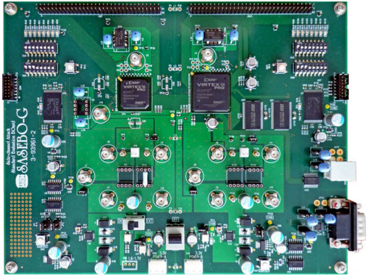
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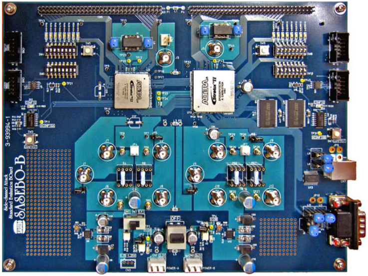

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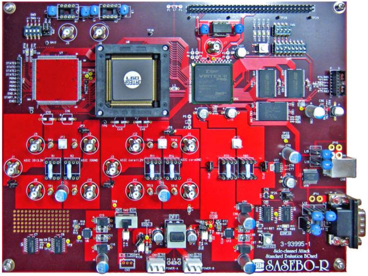

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l'union fait la force
Eendracht maakt macht
Einigkeit macht stark

Notations

- secret key k^* deterministic but unknown
- m independent measurements $\mathbf{x} = (x_1, ..., x_m)$ and independent and uniformly distributed inputs $\mathbf{t} = (t_1, ..., t_m)$
- leakage model $\mathbf{y}(k) = \varphi(f(k, \mathbf{t}))$, where φ is a device specific leakage function and f maps the inputs to an intermediate algorithmic state
- $\mathbf{x} = \mathbf{y}(k^*) + \mathbf{n}$ with independent noise $\mathbf{n}$

Perceived information

Idea [Renauld et al., 2011]

- Metric quantifying degraded leakage models
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- Generalization of mutual information

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Ideal case

- the distribution $\mathbb{P}$ is known
- PI is MI

$$MI(K;X,T) = H(K) + \sum_k \mathbb{P}(k) \sum_t \mathbb{P}(t) \sum_x \mathbb{P}(x|t,k) \log_2 \mathbb{P}(k|t,x)$$

Perceived information

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Real case

- the distribution $\mathbb{P}$ is unknown
- test a profiled model $\hat{\mathbb{P}}$ against an online estimated model $\tilde{\mathbb{P}}$

$$\hat{PI}(K;X,T) = H(K) + \sum_k \mathbb{P}(k) \sum_t \mathbb{P}(t) \sum_x \tilde{\mathbb{P}}(x|t,k) \log_2 \hat{\mathbb{P}}(k|t,x)$$

Outlines

Maximum a posteriori probability

MAP

The optimal distinguishing rule is given by the *maximum a posteriori probability (MAP)* rule

$$\mathcal{D}(\mathbf{x}, \mathbf{t}) = \arg\max_{k} \ \mathbb{P}(k|\mathbf{x}, \mathbf{t}).$$

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With the help of Bayes' rule...

$$\mathbb{P}(k|\mathbf{x}, \mathbf{t}) = \frac{\mathbb{P}(\mathbf{x}|k, \mathbf{t}) \cdot \mathbb{P}(k)}{\mathbb{P}(\mathbf{x}|\mathbf{t})} = \frac{\mathbb{P}(\mathbf{x}|k, \mathbf{t}) \cdot \mathbb{P}(k)}{\sum_k \mathbb{P}(k)\mathbb{P}(\mathbf{x}|\mathbf{t}, k)}.$$

Relation between MAP and PI

- Profiling scenario
- Profiled model $\hat{\mathbb{P}}$, model $\tilde{\mathbb{P}}$ estimated online on-the-fly

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- Profiled model $\hat{\mathbb{P}}$, model $\tilde{\mathbb{P}}$ estimated online on-the-fly
- $\hat{\mathbb{P}}(k|\mathbf{x},\mathbf{t}) \propto \prod_{i=1}^{m} \hat{\mathbb{P}}(k|x_i,t_i)$

We start by maximizing MAP:

$$\arg\max_k \hat{\mathbb{P}}(k|\mathbf{x},\mathbf{t}) = \arg\max_k \prod_{i=1}^{m} \hat{\mathbb{P}}(k|x_i,t_i)$$
$$= \arg\max_k \prod_{x,t} \hat{\mathbb{P}}(k|x,t)^{m\tilde{\mathbb{P}}_k(x,t)},$$

where $\tilde{\mathbb{P}}_k(x,t) = \tilde{\mathbb{P}}(x,t|k)$ is the "counting" estimation (online) of x and t that depends on k. Now taking the log_2 gives

$$= \arg\max_k \sum_{x,t} \tilde{\mathbb{P}}_k(x,t) \log_2 \hat{\mathbb{P}}(k|x,t)$$

Relation between MAP and PI (cont'd)

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$$= \arg\max_k \sum_{t} \tilde{\mathbb{P}}(t) \sum_{x} \tilde{\mathbb{P}}(x|t,k) \log_2 \hat{\mathbb{P}}(k|x,t)$$

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Taking the average over k and adding $H(K)$ gives $\hat{PI}(K;X,T) =$

$$H(K) + \sum_k \mathbb{P}(k) \sum_t \tilde{\mathbb{P}}(t) \sum_x \tilde{\mathbb{P}}(x|t,k) \log_2 \hat{\mathbb{P}}(k|x,t).$$

(except $\tilde{\mathbb{P}}(t)$ vs. $\mathbb{P}(t)$)

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Relation between MAP and PI (cont'd)

PI $\Leftrightarrow$ MAP

$\hat{P}I$ (real case) is the expectation of the MAP over the keys.

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Profiled case

If we have an infinite number of traces to estimate $\tilde{\mathbb{P}} \to \mathbb{P}$ then we recover PI(K;X,T).

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Profiled case

If we have an infinite number of traces to estimate $\tilde{\mathbb{P}} \to \mathbb{P}$ then we recover PI(K;X,T).

Ideal case

If we have an infinite number of traces to estimate $\tilde{\mathbb{P}} \to \mathbb{P}$ and $\hat{\mathbb{P}} \to \mathbb{P}$ then we recover MI(K;X,T).

Assumptions for ML

The leakage model follows the

Markov condition

The leakage $\mathbf{x}$ depends on the secret key k only through the computed model y(k). Thus, we have the Markov chain:

$$(k, t) \to y = \varphi(f(t, k)) \to x.$$

Related to the EIS [Schindler et al., 2005] assumption.

- Markov condition: invariance of conditional probabilities
- EIS assumption: invariance of images under different subkeys

Maximum Likelihood Attack

Maximum Likelihood Attack

Assuming we have $y(k) = \varphi(f(t,k))$ that follows the Markov condition, then the optimal distinguishing rule is given by the maximum likelihood (ML) rule

$$\mathcal{D}(\mathbf{x},\mathbf{t}) = \arg\max_k \ \mathbb{P}(\mathbf{x}|\mathbf{y}).$$

Proven and investigated in [Heuser et al., 2014].

Maximum Likelihood Attack

Similarly, as in the previous derivation we have

$$\arg\max_k \mathbb{P}(\mathbf{x}|\mathbf{y}) = \arg\max_k \prod_{i=1}^{m} \mathbb{P}(x_i|y_i) = \arg\max_k \prod_{x,y} \mathbb{P}(x|y)^{m\tilde{\mathbb{P}}(x,y)}.$$

Taking the $\log_2$ gives us

$$\arg\max_k \sum_{x,y} \tilde{\mathbb{P}}(x,y) \log_2 \mathbb{P}(x|y)$$

Now we add the cross entropy term that does not depend on a key guess k

$$-\sum_{x,y} \tilde{\mathbb{P}}(x,y) \log_2 \mathbb{P}(x).$$

Maximum Likelihood Attack

This results to

$$\arg\max_{k} \sum_{x,y} \tilde{\mathbb{P}}(x,y) \log_2 \frac{\mathbb{P}(y|x)}{\mathbb{P}(y)}.$$

Maximum Likelihood Attack

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In practise...

- $\mathbb{P}$ is most likely not known perfectly by the attacker
- either estimated offline by $\hat{\mathbb{P}}$
- or online on-the-fly $\tilde{\mathbb{P}}$

Maximum Likelihood Attack

Profiled

$\mathbb{P}$ is estimated offline $\hat{\mathbb{P}}$ on a training device

$$\arg\max_k \sum_{x,y} \tilde{\mathbb{P}}(x,y) \log_2 \frac{\hat{\mathbb{P}}(y|x)}{\hat{\mathbb{P}}(y)},$$

which is the *template attack* [Chari et al., 2002].

Distinguisher resulting from the MAP with

- A priori knowledge on the key distribution
- Markov condition

Maximum Likelihood Attack

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Non-Profiled

$\mathbb{P}$ is estimated online $\tilde{\mathbb{P}}$ on a the device under attack

$$\arg\max_k \sum_{x,y} \tilde{\mathbb{P}}(x,y) \log_2 \frac{\tilde{\mathbb{P}}(y|x)}{\tilde{\mathbb{P}}(y)},$$

which gives the Mutual Information Analysis [Gierlichs et al., 2008].

Outlines

Believing or seeing?

Should probabilities be considered as precise as possible?

- Many recent works (e.g., [Veyrat-Charvillon and Standaert, 2009]) showed that using kernel estimation is more efficient than using histograms
- Accordingly, if $\mathbb{P}(Y)$ is known, should it be used instead of $\tilde{\mathbb{P}}(Y)$ and $\hat{\mathbb{P}}(Y)$?

$$\arg\max_k \sum_{x,y} \tilde{\mathbb{P}}(y)\tilde{\mathbb{P}}(x|y) \log_2 \frac{\hat{\mathbb{P}}(y|x)}{\hat{\mathbb{P}}(y)}$$

$$\arg\max_k \sum_{x,y} \tilde{\mathbb{P}}(y)\tilde{\mathbb{P}}(x|y) \log_2 \frac{\tilde{\mathbb{P}}(y|x)}{\tilde{\mathbb{P}}(y)}$$

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Believing or seeing?

Simple scenario

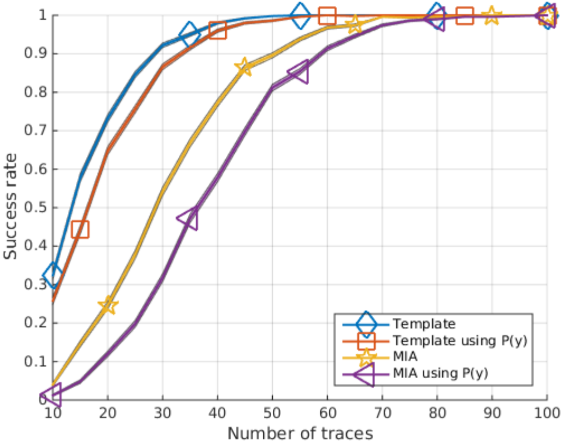
$$X = Y(k^*) + N,$$
$$Y(k) = HW(Sbox(T \oplus k))$$

As Y follows a binomial distribution with parameters $(n, 1/2)$, we have

$$\mathbb{P}(Y) = \{1/256, 8/256, 28/256, 56/256, 28/256, 8/256, 1/256\}.$$

- Template attack: replace $\tilde{\mathbb{P}}(Y)$ and $\hat{\mathbb{P}}(Y)$ by $\mathbb{P}(Y)$
- MIA: replace: $\tilde{\mathbb{P}}(Y)$ by $\mathbb{P}(Y)$

Believing or seeing?



Outlines

Conclusion

- PI is the expectation of the MAP over the keys
- ML is a simple alternative to MAP (with no penalty if keys are uniform)
- Maximum likelihood to recover
  - template attack when probabilities are estimated offline ($\hat{\mathbb{P}}$)
  - MIA when probabilities are estimated online on-the-fly ($\tilde{\mathbb{P}}$)
- All attacks work by "testing" a model (estimated offline or "on-the-fly") against fresh samples
- $\mathbb{P}(Y)$ should be estimated instead of using its theoretical value

Thank you!

Questions?

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