Integration of Fuzzy Structural Information in Deformable Models

Olivier Colliot[*]**, Oscar Camara, Isabelle Bloch**
Ecole Nationale Supérieure des Télécommunications,
Département TSI, CNRS UMR 5141
46, rue Barrault, 75634 Paris Cedex 13, France
{Olivier.Colliot, Oscar.Camara, Isabelle.Bloch}@enst.fr

Abstract

Our purpose is to integrate structural information, such as spatial relations, in variational image processing techniques called deformable models. Spatial relations are represented as fuzzy subsets of the 3D image space. After describing representation and fusion methods for spatial relations, we present several approaches to integrate them in deformable models, deriving new types of external forces from fuzzy sets. Finally, the method is illustrated on a brain structure segmentation application.

Keywords: Spatial relations, deformable models, image segmentation, brain imaging.

1 Introduction

Spatial relations constitute the basic elements contained in linguistic descriptions of spatial configurations. These relations express the spatial arrangement of objects with respect to the others. Spatial relations are usually classified into different types including topological, distances and directional relations [16]. Their importance has been highlighted in many domains related to computer science and engineering, such as artificial intelligence, computational linguistics, geographic information systems or autonomous robotics.

The aim of this paper is to use such spatial relations for segmentation and recognition of objects in images. In image processing and pattern recognition, these relations can be considered as a subtype of structural knowledge which opposes to numerical information such as grey level or texture. Their ability to describe scenes make them potentially useful for a wide range of imaging tasks, as long as they concern structured scenes, i.e. scenes in which objects share stable relations. Such scenes can be found for example in aerial imaging [9, 10], face recognition [4] or medical imaging. The human brain is a typical case of structured scene in which brain structures share stable relations [7]. Segmentation of brain structures will be considered as the underlying example in this paper.

The fuzzy set theory is well suited to the representation of spatial relations because it provides a common representation framework for heterogeneous information and it has the ability to represent the imprecision induced by image processing operations as well as by the relations themselves.

Spatial relations have been used in a relatively small number of imaging applications (e.g. [3, 4, 9, 10, 14]). Moreover, in all these applications, the relations are used for high-level tasks (i.e. recognition), the low-level processing (i.e. segmentation) being done with classical techniques based only on numerical information. On the contrary, we be-

[*]Current address: McConnell Brain Imaging Center, Montreal Neurological Institute, McGill University, 3801 University, Montreal, Québec, H3A 2B4, Canada, colliot@bic.mni.mcgill.ca, Telephone: +1 514 398 8841

lieve that spatial relations could be of great help to find the object contours and that they should be directly integrated in segmentation techniques. We chose to integrate them in deformable models [8] which are effective when dealing with noisy images and objects with imperfect boundaries and which constitute an appropriate framework to merge heterogeneous information.

The paper is organized as follows. In Section 2, we explain how structural information can be modeled using fuzzy sets. Afterwards, in Section 3, the integration of spatial relations and deformable models is detailed. Finally, in Section 4, we apply the proposed technique to the segmentation of brain structures in Magnetic Resonance Images (MRI).

2 Representation of structural information using fuzzy sets

Fuzzy sets constitute an appealing framework to represent spatial relations. First, some imprecision can be introduced by the imperfections of the image processing. Then, some relations, corresponding to linguistic expressions, can be intrinsically imprecise. The satisfaction of a given relation will thus be defined as a matter of degree rather than in an "all-or-nothing" manner. Given a relation with respect to a reference fuzzy object A, two types of questions can be addressed:

- compute to which degree a target object B fulfills this relation;
- find the points of the space where this relation is satisfied.

The first one has been addressed for a wide range of relations including adjacency [2], distances [2], directions [11, 12] and symmetries [6]. In this work, we will consider the second approach as our aim is to make the deformable model evolve towards the points where the relation is satisfied. We recall here how, using this approach, distance and directional relations can be represented by a fuzzy set in the 3D space and how these relations can be combined using fusion operations.

2.1 Distances

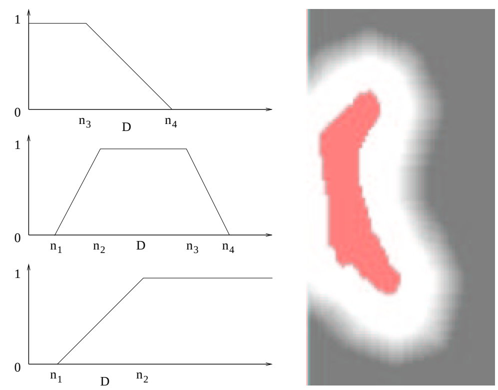
The approach proposed in [2] considers the case of distance relations such "at a distance equal to n", "near" or "far from" a reference object. Its principle is to define a fuzzy subset of the 3D space $\mathcal{S}$ representing in each point the degree of satisfaction of the relation.

The semantics of a relation of this type can be represented by a fuzzy interval μ_n of the set of distances $\mathbb{R}^+$. One can choose a trapezoidal shape for μ_n and then values $0 \leq n_1 \leq n_2 \leq n_3 \leq n_4$ in $\mathbb{R}^+$ are points such that the kernel of μ_n is $[n_2, n_3]$ and its support is $[n_1, n_4]$ (see Figure 1(a)). The following fuzzy structuring elements are then defined:

$$\nu_1(P) = \begin{cases} 1 - \mu_n(d_E(P,0)) & \text{if } d_E(P,0) \leq n_2 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$\nu_2(P) = \begin{cases} 1 & \text{if } d_E(P,0) \leq n_3 \\ \mu_n(d_E(P,0)) & \text{otherwise} \end{cases} \quad (2)$$

where d_E is the Euclidean distance in $\mathcal{S}$, P a point in $\mathcal{S}$, and $d_E(P,0)$ is the Euclidean distance between P and the center of the structuring element.



(a) (b)

Figure 1: Representation of distance relations. (a) Fuzzy intervals representing relations "near", "at a distance equal to D" and "far from" (from top to bottom). (b) Spatial fuzzy set representing the relation "near". Bright areas correspond to high values.

The fuzzy set representing the distance relation with respect to A is then defined as :

$$\mu_d = t\big[D_{\nu_2}(\mu_A), 1 - D_{\nu_1}(\mu_A)\big], \quad (3)$$

where $D_{\nu_i}(\mu_A), i \in \{1, 2\}$ is the fuzzy dilation of μ_A by the structuring element ν_i. The parameters n_1, n_2, n_3, n_4 will be chosen according to the relation under consideration. Figure 1 presents some examples of fuzzy intervals and a corresponding fuzzy set.

However, fuzzy dilations may be computationally expensive and in the case of a crisp reference object one can use μ_n as a look-up table, composed with a distance map to object A.

2.2 Directions

We consider the case of 6 relations corresponding to the 3 main directions of the 3D space: "left", "right", "above", "below", "in front of", "behind". Again, each relation is represented by a fuzzy set in the 3D space $\mathcal{S}$, called a "fuzzy landscape", following the approach proposed in [2].

Let $\mathbf{u}_{\alpha_1,\alpha_2}$ ($\alpha_1 \in]-\pi/2, \pi/2]$ and $\alpha_2 \in [0, 2\pi[$) be a unit vector corresponding to the relation under consideration $\alpha = (\alpha_1, \alpha_2)$, P a point in $\mathcal{S}$, Q a point of the reference object A and $\beta(P, Q)$ the angle between vectors $\mathbf{QP}$ and $\mathbf{u}_{\alpha_1,\alpha_2}$, computed in $[0, \pi]$:

$$\beta(P, Q) = \arccos\left[\frac{\mathbf{QP} \cdot \mathbf{u}_{\alpha_1,\alpha_2}}{\|\mathbf{QP}\|}\right] \text{ and } \beta(P, P) = 0 \tag{4}$$

Then, for each point P, we compute:

$$\beta_{min}(P) = \min_{Q \in A} \beta(P, Q) \tag{5}$$

The "fuzzy landscape" is then defined as:

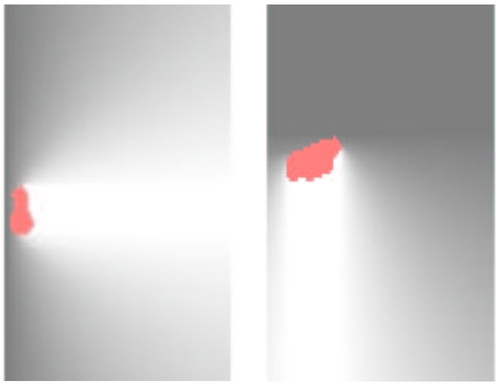
$$\mu_\alpha(P) = f(\beta_{min}(P)) \tag{6}$$

where f is a decreasing function from $[0, \pi]$ in $[0, 1]$, e.g. $f(\theta) = \max[0, 1 - (2/\pi)\theta]$. If A is a fuzzy object, it can be defined as:

$$\mu_\alpha(P) = \max_{Q \in supp(A)} t[\mu_A(Q), f(\beta(P, Q))] \tag{7}$$

where t is a t-norm and $supp(A)$ is the support of A. Finally, one can show that $\mu_\alpha(P)$ corresponds to a fuzzy dilation of the reference object by the following structuring element:

$$\forall P \in \mathcal{S}, \nu(P) = f(\beta(O, P)) \tag{8}$$



(a) (b)

Figure 2: Representation of directional relations. Fuzzy landscapes corresponding to: (a) "to the right of ", (b) "below".

where O is the center of the structuring element.

Figure 2 presents two examples of fuzzy landscapes for directional relations.

2.3 Fusion of relations

An additional advantage of representing a relation with a spatial fuzzy set is that relations can be combined using fusion operations. Examples of combinations include: "to the right or below", "to the left and near" ... From two fuzzy sets representing two relations R_1 and R_2, one can compute a set R corresponding to their combination using a fusion operator F: $\mu_R(P) = F(\mu_{R_1}(P), \mu_{R_2}(P))$. A review of fusion operators can be found in [1]. In the following, we will use t-norms operators for conjunctive fusion and t-conorms for disjunctive combination. Figure 3 presents an example of a fusioned relation.

3 Integration in a deformable model

3.1 The deformable model paradigm

Deformable models are contours or surfaces evolving within an image from a starting point to a final state that should correspond to the targeted object (e.g. the object we want to segment). Two types of information usually drive the evolution: a data term that attracts

Figure 3: Fusion of the two relations from Figure 2 with the t-norm "product".

the model towards the edges of the image and a regularization term that forces the model to stay smooth and regular. One can distinguish two families of deformable models [17]: parametric and geometric ones. Here, we will consider the first case, in which the evolution can be described for example by the following dynamic force equation:

$$\gamma \frac{\partial \mathbf{X}}{\partial t} = \mathbf{F}_{int}(\mathbf{X}) + \mathbf{F}_{ext}(\mathbf{X}) \tag{9}$$

where $\mathbf{X}$ is the deformable contour or surface, $\mathbf{F}_{int}$ is the internal force that specifies the regularity of the surface and $\mathbf{F}_{ext}$ is the external force that drives the surface towards image edges.

Many different choices can be made concerning either the parametric or discrete representation of the contour or surface, the regularization term or the external force. We will not present these aspects here, concentrating only on the integration of spatial relations in the deformable model. Details can be found in reviews on deformable models [17, 13].

3.2 Proposed approach

Deformable models provide a convenient framework to merge different types of information, by combining terms in the evolution scheme. A considerable amount of research has been carried out to introduce shape constraints in deformable models. However, to our knowledge, structural knowledge has nearly never been introduced in this context.

We propose to introduce spatial relations in the evolution scheme, replacing the external force $\mathbf{F}_{ext}$ in Equation 9 with a force describing both edge information and structural constraints:

$$\mathbf{F}_{ext} = \lambda \mathbf{F}_C + \nu \mathbf{F}_R \tag{10}$$

where $\mathbf{F}_C$ is a classical data term that will drive the model towards the edges, $\mathbf{F}_R$ is a force associated to spatial relations and λ and ν are weighting coefficients.

Let R be a fuzzy set representing a spatial relation and μ_R its membership function. The force $\mathbf{F}_R$ should constrain the model to stay in regions where the relation is fulfilled and then be directed towards high values of μ_R. When the relation is completely satisfied, the model should be only driven by edge information, i.e. $\mathbf{F}_R$ should be zero in the kernel of R. The less the relation is satisfied the higher the modulus of the force should be, thus we impose it to be proportional to $(1 - \mu_R)$. Finally, the computation time for the force should be reasonable. The following describes construction methods for external forces that fulfill these properties.

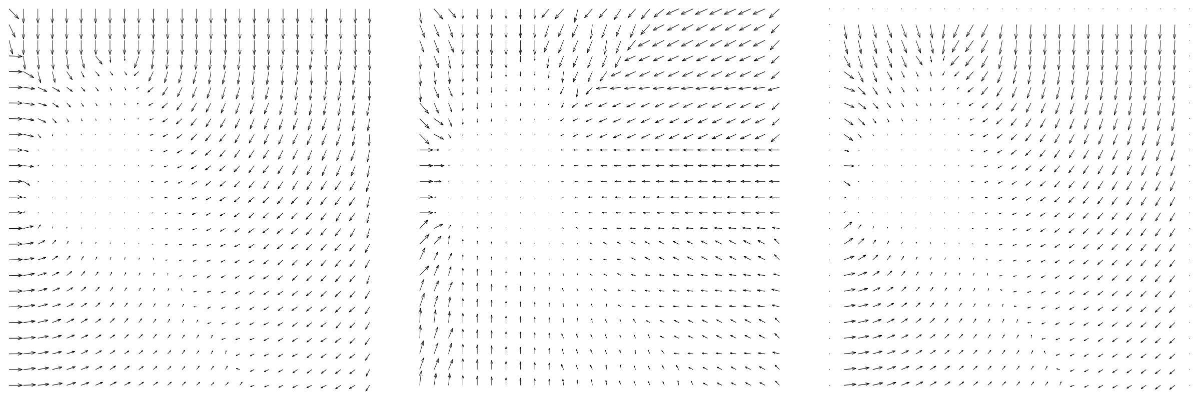
Using the fuzzy set as a potential At first sight, one could think that an energy potential could be derived directly from the fuzzy set, e.g. $P_R = 1 - \mu_R$, leading to a potential force $\mathbf{F}_R = -\nabla P_R$. However, such a force would obviously have zero values outside the support of R, which is highly undesirable as the relation is completely unsatisfied in this region. This can be solved by adding to the potential the distance from the support, then defining:

$$P_R^1(P) = 1 - \mu_R(P) + d_{supp(R)}(P) \tag{11}$$

where $d_{supp(R)}$ is the distance from the support of R. With the following normalization, we obtain a force satisfying the required properties:

$$\mathbf{F}_R^1(P) = -(1 - \mu_R(P)) \frac{\nabla P_R^1(P)}{\|\nabla P_R^1(P)\|} \tag{12}$$

An example of external force computed using this approach is shown in Figure 4(a).



(a) (b) (c)

Figure 4: External force corresponding to the spatial relation presented in Figure 3: (a) computed using the first method $\mathbf{F}_R^1$ (for visualization purposes a 1/3 under-sampling has been performed), (b) with the second method $\mathbf{F}_R^2$, (c) with the third method $\mathbf{F}_R^3$.

Using a distance potential force Distance potential forces [5], defining a potential as a function of a distance map to a binary edge detector, provide a large attraction range, which is of interest in our case as we want a non-zero force everywhere outside the kernel of R. Nevertheless, if we want to replace the edge map with the fuzzy set, we need to use a fuzzy distance instead of a classical one. For example, good properties would be obtained with the fuzzy morphological distance defined as: $d_\nu = 1 - D_\nu(\mu_R)$ where ν is a structuring element with radial symmetry: $\nu(x, y, z) = 1 - \frac{\sqrt{x^2+y^2+z^2}}{k}$ and k is the size of this element. A potential would then be defined as $P_R(P) = g(d_\nu(P))$where g is a non-decreasing function, e.g. $g(x) = -1/x$. However, the morphological distance is computationally expensive. For 3D applications, we recommend to replace it with a classical distance such as the distance to the kernel of R:

$$P_R^2(P) = g(d_{Ker(R)}(P)) \qquad (13)$$

where $d_{Ker(R)}(P)$ is a distance map to the kernel of R. The corresponding force, denoted by $\mathbf{F}_R^2$, is computed using the same formula as in Equation 12. Figure 4(b) presents a force constructed using this equation.

Using a gradient diffusion technique Using a gradient vector diffusion technique also allows to have a wide attraction range. The Gradient Vector Flow (GVF), introduced by Xu et al. [17], computes a smooth vector field while being close to the original in the regions where it has high values. Here, we replace the edge map with our fuzzy set μ_R in the original GVF formulation:

$$\begin{cases} \frac{\partial v}{\partial t} = c\nabla^2 v - \|\nabla\mu_R\|^2(v - \nabla\mu_R) \\ v(P, 0) = \nabla\mu_R(P) \end{cases} \qquad (14)$$

The first equation is a combination of a diffusion term that will produce a smooth vector field and a data term that encourages v to stay close to $\nabla\mu_R$. In regions where $\|\nabla\mu_R\|$ is low, the diffusion term will prevail. In particular, inside the kernel and outside the support of R, only diffusion will occur, giving a non-zero force. However, as we want the force to be zero in the support, we will use the following normalization:

$$\mathbf{F}_R^3 = (1 - \mu_R)\frac{\mathbf{u}}{\|\mathbf{u}\|} \qquad (15)$$

where $\mathbf{u}$ is the GVF. An example is shown in Figure 4(c).

3.3 Discussion

The three proposed external forces have good properties but, although they share a similar behavior, they are not equivalent. In the particular case of a fuzzy set with local maxima outside its kernel, $\mathbf{F}_R^1$ and $\mathbf{F}_R^3$ would be

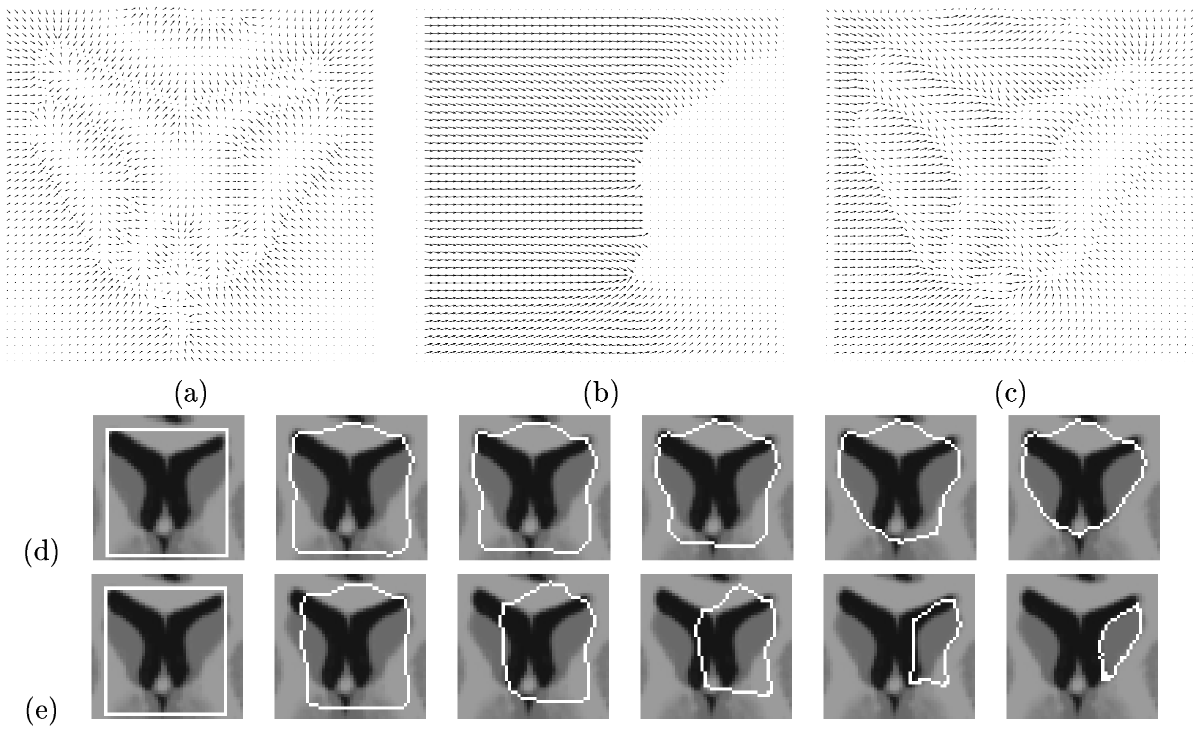


Figure 5: Basic example of deformable model driven by structural information. (a) Classical data term. (b) Force $\mathbf{F}_R^3$ representing the spatial relation "to the right of the ventricle (in black on the image)". (c) Combination of the 2 previous forces using Equation 10. (d) Evolution: using only the data term and starting from the white rectangle on the left image, the deformable model is attracted by the first strong edges that it encounters. (e) Using the combination of both the data term and the spatial relation, it is able to converge to the caudate nucleus (in grey).

directed towards these maxima, whereas $\mathbf{F}_R^2$ always points towards the kernel and should probably not be used in that case. $\mathbf{F}_R^1$ is always directed orthogonally to the isolevels of R, while $\mathbf{F}_R^3$ nearly fulfills this property but introduces an additional regularization. The computational cost of $\mathbf{F}_R^1$ and $\mathbf{F}_R^2$ is very low (5 seconds for a 128x128x124 image on a PC Pentium III 1Ghz). The computation time is higher for $\mathbf{F}_R^3$ (3 minutes), while staying reasonable, this being the price for regularization. It is quite difficult to recommend a particular force and in our experiments they have led to similar results. An additional comment concerns the combination scheme proposed in Equation 10. It could also be possible to use the fuzzy set as a weighting function for the data term, thus not taking it into account where the relation is completely unfulfilled:

$$\mathbf{F}_{ext} = \lambda \mu_R \mathbf{F}_C + \nu \mathbf{F}_R \qquad (16)$$

To conclude this section, let us give a brief comparison with other approaches. Pitiot et al. [15] proposed a deformable model driven by knowledge-based constraints which include shape but also distance information. Distance constraints are represented with a new force introduced in the evolution scheme. The main difference with our approach, proposed independently, is that we consider different types of spatial relations, represented in the common framework of fuzzy sets.

Xu et al. [18] introduced constraints represented by fuzzy sets in a deformable model. However, these fuzzy sets convey numerical information, namely grey-level classes (of grey and white matter in a brain reconstruction application) derived from a fuzzy classification, instead of structural knowledge. The authors used a balloon force the direction of which is orthogonal to the deformable contour, its

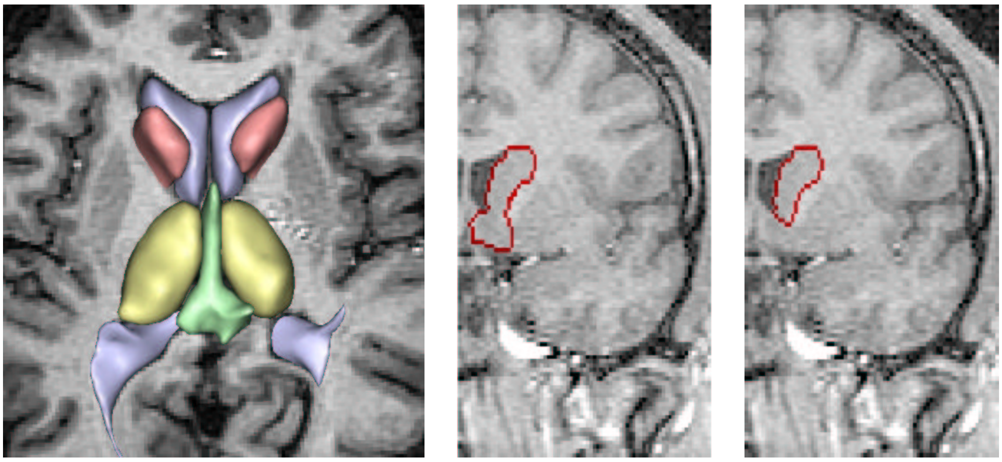


Figure 6: Results obtained for the lateral ventricles, the third ventricle, the caudate nuclei and the thalami. (a) 3D rendering superimposed on an axial slice. (b) Segmentation of the caudate nucleus: without a spatial relation, the model progresses beyond the boundary of the object. (c) This problem is solved by forcing the model to stay "near and to the right to the lateral ventricle". 3D images have been visualized using the Anatomist software (www.anatomist.info).

magnitude being computed from the membership functions of the fuzzy sets. In our case, we did not consider the use of a pressure force because we would have no way to determine the direction of evolution based on the value of membership function.

4 Application to brain structures segmentation in MRI

4.1 A basic example

The example presented in Figure 5 is a synthetic 2D image representing a portion of the brain (extracted from the BrainWeb database[1]) and the objective is to segment the caudate nucleus (in grey). When only a data term and a regularization term are considered (Figure 5(a)), the model will be attracted by the first strong edges that it encounters. On the contrary, when adding a spatial relation term (Figure 5(b)), the deformable contour avoids objects that do not fulfill the relation to converge towards the targeted one. This illustrates that spatial relation terms allow us to initialize the model far from the targeted objects.

[1]http://www.bic.mni.mcgill.ca/brainweb/

4.2 Subcortical brain structures segmentation

We applied our methodology to the segmentation of subcortical brain structures in Magnetic Resonance Images (MRI), which is known to be a challenging problem due in particular to the low-constrast and the lack of strong edges between some structures. A description of the brain anatomy is introduced using spatial relations between brain structures. These relations are extensively used, in particular to constrain a 3D deformable model. As illustrated in Figure 6, good segmentation results are obtained and spatial relations have proved useful to prevent the model to go beyond the limit of structures with weak boundaries. All implementation details can be found in [7].

5 Conclusion

We proposed an approach to integrate fuzzy structural knowledge in a deformable model. This knowledge is made of spatial relations such as distances and directions but our approach can potentially handle any type of information that can be represented by a spatial fuzzy set. Its principle is to derive, from the fuzzy set, a new external force that will be introduced in the evolution scheme of the model. As illustrated by examples, this method can substantially improve the segmentation of objects with weak boundaries. Main perspectives of this work are the combination of this structural knowledge with shape constraints in a deformable model and the simultaneous evolution of several models.

References

[1] I. Bloch. Information combination operators for data fusion: A comparative review with classification. *IEEE Transactions on Systems, Man, and Cybernetics*, 26(1):52–67, 1996.

[2] I. Bloch. Mathematical morphology and spatial relationships: Quantitative, semi-quantitative and symbolic settings. In L. Sztandera and P. Matsakis, editors, *Applying Soft Computing in Defining Spatial Relationships*, pages 63–98. Physica Verlag, Springer, 2002.

[3] I Bloch, T. Géraud, and H. Maître. Representation and fusion of heterogeneous fuzzy information in the 3D space for model-based structural recognition - application to 3D brain imaging. *Artificial Intelligence*, 148:141–175, August 2003.

[4] R.M. Cesar, E. Bengoetxea, and I. Bloch. Inexact graph matching using stochastic optimization techniques for facial feature recognition. In *Proc. ICPR 2002*, Québec, Canada, 2002.

[5] L.D. Cohen and I. Cohen. Finite element methods for active contour models and balloons for 2D and 3D images. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 15(11):1131–1147, 1993.

[6] O. Colliot, I. Bloch, and A.V. Tuzikov. Characterization of approximate plane symmetries for 3D fuzzy objects. In *Proc. IPMU 2002*, volume 3, pages 1749–1756, Annecy, France, 2002.

[7] O. Colliot, O.Camara, R. Dewynter, and I.Bloch. Description of brain internal structures by means of spatial relations for MR image segmentation. In *Proc. SPIE 2004 Medical Imaging*, 2004.

[8] M. Kass, A. Witkin, and D. Terzopoulos. Snakes : Active contour models. *International Journal of Computer Vision*, pages 321–331, 1987.

[9] J.M. Keller, P. Gader, and X. Wang. Ladar scene description using fuzzy morphology and rules. In *Proceedings. IEEE Workshop on Computer Vision Beyond the Visible Spectrum: Methods and Applications*, 1999.

[10] F. Le Ber and L. Mangelick. A Formal Representation of Landscape Spatial Patterns to Analyze Satellite Images. *AI Applications*, 12(1-3):51–59, 1998.

[11] P. Matsakis and L. Wendling. A new way to represent the relative position between areal objects. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 21(7):634–643, July 1999.

[12] K. Miyajima and A. Ralescu. Spatial organization in 2D segmented images: representation and recognition of primitive spatial relations. *Fuzzy Sets and Systems*, 65(2-3):225–236, August 1994.

[13] J. Montagnat, H. Delingette, and N. Ayache. A review of deformable surfaces: topology, geometry and deformation. *Image and Vision Computing*, 19:1023–1040, 2001.

[14] A. Perchant and I. Bloch. A new definition for fuzzy attributed graph homomorphism with application to structural shape recognition in brain imaging. In *Proc. IEEE IMTC 1999*, volume 3, pages 1801–1806, 1999.

[15] A. Pitiot, H. Delingette, N. Ayache, and P.M. Thompson. Expert knowledge guided segmentation system for brain MRI. In *Proc. MICCAI 2003*, pages 644–652, Montreal, 2003.

[16] L. Vieu. Spatial representation and reasoning in artificial intelligence. In *Spatial and Temporal Reasoning*, pages 5–41. Dordrecht, Kluwer, 1997.

[17] C. Xu, D.L. Pham, and J.L. Prince. Medical image segmentation using deformable models. In J.M. Fitzpatrick and M. Sonka, editors, *Handbook of Medical Imaging*, volume 2, pages 129–174. SPIE Press, 2000.

[18] C. Xu, D.L. Pham, M.E. Rettmann, D.N. Yu, and J.L. Prince. Reconstruction of the human cerebral cortex from magnetic resonance images. *IEEE Transactions on Medical Imaging*, 18(6):467–480, June 1999.