FUZZY RELATIVE POSITION BETWEEN OBJECTS IN IMAGES: A MORPHOLOGICAL APPROACH

Isabelle Bloch

Ecole Nationale Supérieure des Télécommunications, département Images,
CNRS URA 820, 46 rue Barrault, 75634 Paris Cedex 13, France
Tél: 33 1 45 81 75 85, Fax: 33 1 45 81 37 94, E-mail : bloch@ima.enst.fr

ABSTRACT

In order to cope with the ambiguity of spatial relative position concepts, we propose new definitions of the relative position between two objects in the fuzzy set framework, which are based on fuzzy pattern matching approaches. They have good properties, are flexible, fit the intuition, and they can be used for structural pattern recognition under imprecision. Moreover, they apply also in 3D, and for fuzzy image objects.

1. INTRODUCTION

The spatial arrangements of objects in images constitute an important information for recognition and interpretation tasks, in particular when the objects are embedded in a complex environment, like in medical images (especially brain images), or satellite and aerial images. A part of the relationships between objects can be described in terms of relative position, like "left to". This paper addresses the problem of defining such relationships. It should be noted that such concepts are highly ambiguous, and they defy precise definitions, although human beings may have a rather intuitive understanding of them. In particular, any "all-or-nothing" definition may lead to unsatisfactory results in several situations, even of moderate complexity (see examples of Fig. 1).

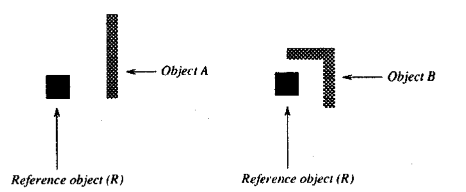

Figure 1: Two examples where the relative position of objects with respect to the reference object is difficult to define in a "all-or-nothing" manner.

Therefore, relative position concepts may find a better understanding in the framework of fuzzy sets, as fuzzy relationships. It is possible to propose flexible definitions, which fit the intuition and may include subjective aspects, depending on the application and on the requirements of the user.

This problem has already been addressed in the literature and we will first review the proposed methods, mainly based on angle computations. To our point of view, none of these methods include real information on object shapes. Therefore, we propose two original methods, based on a fuzzy pattern matching approach. The first one is also based on angle computation. The second one is really morphological. These definitions will be compared in a qualitative way on the examples of Fig. 1, and more rigorously by studying their properties. In particular, they will be examined under the light of possible generalization, to 3D objects on one hand, and to fuzzy objects on the other. Indeed, the representation of image regions as spatial fuzzy sets is very useful in order to take into account the imprecision inherent to the images. As a conclusion, we will provide some hints on the foreseen application in fuzzy structural pattern recognition.

In the sequel, we will deal with finite discrete images.

2. ANGLE BASED METHODS

To our knowledge, existing methods for defining fuzzy relative spatial position all rely on angle measurements between points of the two objects of interest [6], [5], and concern 2D objects. A fuzzy relationships is defined as a fuzzy set, and the adequation between the relation and the angle mesurements is evaluated, according to three methods which will be described below.

2.1. Fuzzy relations describing relative position

In [6], [5], angle is computed between the segment joining two points a and b and the x-axis of the coordinate frame. This angle, denoted by $\theta(a,b)$, takes values in $[-\pi,\pi]$. A relative position relationship is then defined as a fuzzy set depending on θ. Fig. 2 illustrates the four relations "left", "right", "above" and "below", defined in [6], as $\cos^2\theta$ and $\sin^2\theta$ functions. Other functions are possible: [5] use trapeze shaped membership functions, for the same 4 relations. The membership functions for these relations will be denoted by μ_{left}, μ_{right}, μ_{above}, and μ_{below}, and are functions from $[-\pi,\pi]$ into [0,1].

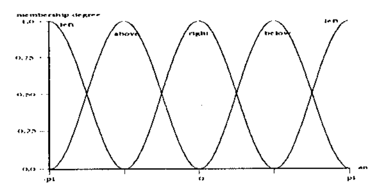

Figure 2: Definition of fuzzy relations representing relative position.

2.2. Centroid method

A first simple solution to evaluate a fuzzy relationship between two objects consists in representing each object by a characteristic point. This point is chosen as the object centroid in [5]. Let c_R and c_A denote the centroids of objects R and A. The degree of satisfaction of the proposition "A is to the right of R" is then defined as:

$$\mu^R_{right}(A) = \mu_{right}(\theta(c_R, c_A)). \quad (1)$$

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2.3. Aggregation method

An aggregation method has been proposed in [5], which uses all points of both objects instead of only one characteristic point. For any couple of points i in R and j in A, the angle $\theta(i,j)$ is computed, and the corresponding membership value for a relation "rel" ("rel" being one of the 4 considered relations) is computed as previously: $\mu_{ij} = \mu_{rel}(\theta(i,j))$. All these values are then aggregated. The aggregation operator suggested in [5] is a weighted mean:

$$\mu^R_{rel}(A) = \left[\sum_{i\in R}\sum_{j\in A} w_{ij}\mu^p_{ij}\right]^{1/p}, \qquad (2)$$

where w_{ij} are weights whose sum is equal to 1.

2.4. Compatibility method

A compatibility method has been proposed in [6]. It consists in defining a fuzzy set in [0,1] representing the compatibility between the normalized angle histogram and the fuzzy relation. More precisely, the angle histogram is computed from the angle between any two points in both objects as defined before, and normalized by the maximum frequency. Let us denote $H^R(A)$ this normalized histogram, for R being the reference object and A the object whose position with respect to R will be evaluated. The compatibility set $\mu_{C(H,\mu_{rel})}$ between $H^R(A)$ and μ_{rel} is defined, for any $u \in [0,1]$, following the extension principle as:

$$\begin{aligned}\mu_{C(H,\mu_{rel})}(u) &= 0 \quad if\ \mu^{-1}_{rel}(u) = \emptyset \qquad (3)\\ &= \sup_{v|u=\mu_{rel}(v)} H^R(A)(v)\ else.\end{aligned}$$

A global evaluation of the position is then provided by the center of gravity of the compatibility fuzzy set. Fig. 3 presents the angle histograms for the two examples of Fig. 1. The compatibility fuzzy sets for 2 relative positions are presented on Fig. 4 for object B with respect to reference object R (defined on Fig. 1).

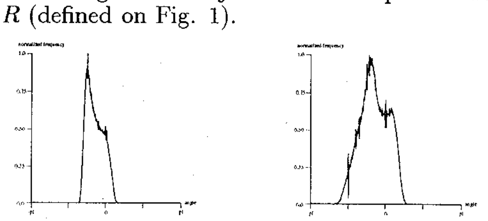
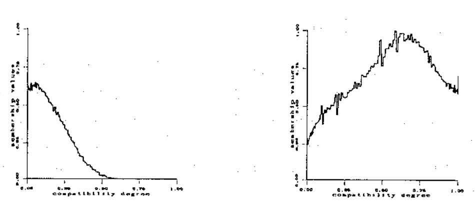


Figure 3: Angle histograms for the two examples of Fig. 1. Left: object A w.r.t. reference object R; right: object B w.r.t reference object R.



"below" "right"

Figure 4: Compatibility between 2 relations and the angle histogram of object B with respect to reference object R.

2.5. A new method based on fuzzy pattern matching

We propose in this Section another method for evaluating the adequation between the angle histogram and the relation membership functions. The idea is to give both functions an interpretation as possibility distribution and to take into account the morphology of these distributions for evaluating their correspondence. An appropriate tool for this task is the fuzzy pattern matching approach [4]. In this approach, the evaluation of the matching between two possibility distributions consists of two numbers, a necessity degree N (a pessimistic evaluation) and a possibility degree Π (an optimistic evaluation). In our application, they will take the following forms:

$$\Pi^R_{rel}(A) = \sup_{x\in[-\pi,\pi]} i[H^R(A)(x), \mu_{rel}(x)], \qquad (4)$$

$$N^R_{rel}(A) = \inf_{x\in[-\pi,\pi]} u[H^R(A)(x), 1-\mu_{rel}(x)], \qquad (5)$$

where i is a t-norm and u a t-conorm.

The possibility corresponds to a degree of intersection, while the necessity corresponds to a degree of inclusion. They can also be interpreted in terms of fuzzy mathematical morphology, since the possibility is equal to the dilation of $H^R(A)$ by μ_{rel} at origin, while the necessity is equal to the erosion, as shown in [2]. These two interpretations, in terms of set theoretic operations and in terms of morphological ones, explain how the shape of the distributions is taken into account.

3. MORPHOLOGICAL FUZZY PATTERN MATCHING METHOD

In this Section, we propose an original method, which relies on completely different bases in comparison to the previous ones. The aim is to take directly into account morphology and shape of the objects, without intermediary angle histogram computation.

3.1. Principle

The idea is to evaluate the adequation of an object in a fuzzy "landscape" whose membership values of the points correspond to the degree of satisfaction of the spatial relation under examination with respect to the reference object. We make use here of a spatial representation of fuzzy sets, which already proved to be useful in image processing. Here again, a fuzzy pattern matching approach is appropriate. We will call it morphological fuzzy pattern matching approach (MFPM), in opposition to the previous compatibility fuzzy pattern matching approach (CFPM). Let us denote by $\mu_{rel}(R)$ the fuzzy set defined in the image in such a way that area which satisfy to a high degree the relation "rel" with respect to reference object R have high membership values. In other terms, the membership function $\mu_{rel}(R)$ has to be an increasing function of the degree of satisfaction of the relation. It is a spatial fuzzy set (i.e. a function of the image $\mathcal{I}$ into [0,1]) and is directly related to the shape of R. Let us denote by μ_A the characteristic function of the object A, which is a function of $\mathcal{I}$ into [0,1]. If A is a fuzzy set, μ_A will be its membership function. So the following definitions apply for both crisp and fuzzy objects. The evaluation of relative position of A with respect to R is given by the possibility and necessity degrees of the fuzzy pattern matching of μ_A and $\mu_{rel}(R)$:

$$\Pi^R_{rel}(A) = \sup_{x\in\mathcal{I}} i[\mu_{rel}(R)(x), \mu_A(x)], \qquad (6)$$

$$N_{rel}^{R}(A) = \inf_{x \in \mathcal{I}} u[\mu_{rel}(R)(x), 1 - \mu_A(x)]. \quad (7)$$

Again, these values can be interpreted as fuzzy intersection and inclusion on the one hand, and as fuzzy dilation and erosion at origin on the other hand. But this time, the objects are directly involved in the computation and therefore the morphological aspect of this definition concerns the objects themselves and not only their angle histogram.

3.2. Defining $\mu_{rel}(R)$

The problem in this definition relies in the definition of $\mu_{rel}(R)$. The requirements stated above for this fuzzy set are not strong and leave room for a large spectrum of possibilities. We suggest here two ways for defining $\mu_{rel}(R)$, sketched on Fig. 5. The first one consists in choosing one reference point c_R in the object (for instance the centroid or its nearest object point in the direction of interest), and computing for each point a of the image its membership value as a decreasing function of the absolute value of the angle $\theta(c_R, a)$ between the line (c_R, a) and the direction of interest. This is illustrated on Fig. 6 (left). Another possibility consists in defining a band around the object in the desired direction where the membership values are constant. Outside this band, they will depend on angle as in the first solution. This is illustrated on Fig. 6 (right). These two definitions are only examples of what is possible. Other definitions could be proposed, and the flexibility of the approach allows the user to define any membership function according to his application and his requirements.

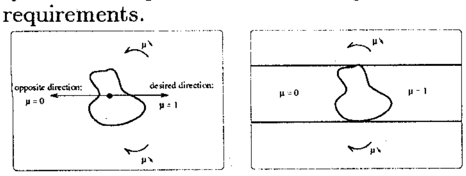


Figure 5: Example of schemes for defining $\mu_{rel}(R)$ for the relative position "right".

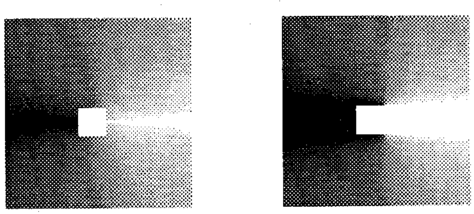


Figure 6: Two possible definitions of $\mu_{rel}(R)$ for the relative position "right" (high grey values correspond to high membership values).

4. EXAMPLES AND COMPARED PROPERTIES

Tables 1 and 2 provide the results obtained with the methods described in the previous sections, for the objects figured on Fig. 1 and for the 4 relations "left", "right", "below", "above". The two new definitions proposed in this paper were motivated by the following limits of previous approaches:

- The centroid method, although robust to small variations of the shapes, is too rough: all information about the objects is lost and it leads to questionable results in case of complex shapes (in particular with strong concavities).

- Methods based on angle histogram are computationally expensive.

- The centroid and aggregation methods are mainly based on a single value to evaluate the relation. The difference between them is that the averaging is made on the object points for the centroid method, while it is applied after angle computation in the aggregation method. On the contrary, the compatibility method presents the advantage to check for an adequation between two fuzzy sets and therefore includes a richer information, but still far from the whole information carried by the objects.

- All three existing methods lake of morphological information.

- These methods are also difficult to generalize to 3D objects, and the cost of histogram based methods would be unreasonable.

Object A with respect to reference object R						
Relation	Centroid	Aggregation	Compatibility	CFPM	MFPM-1	MFPM-2
left	0.000	0.000	0.000	[0.000, 0.000]	[0.000, 0.306]	[0.000, 0.322]
right	0.764	0.726	0.621	[0.371, 0.683]	[0.694, 1.000]	[0.678, 1.000]
below	0.000	0.004	0.054	[0.000, 0.095]	[0.192, 0.576]	[0.137, 0.502]
above	0.236	0.270	0.379	[0.317, 0.629]	[0.423, 0.808]	[0.498, 0.863]

Table 1: Results obtained for the object A of Fig. 1 with respect to reference object R. The relative position is evaluated with the centroid method, the aggregation method, the center of gravity of fuzzy compatibility set, the compatibility fuzzy pattern matching (CFPM) and the morphological fuzzy pattern matching (MFPM-1 and MFPM-2) for the two definitions of $\mu_{rel}(R)$ (see Fig. 6). For FPM approaches, the two given values correspond to necessity and possibility degrees.

Object B with respect to reference object R						
Relation	Centroid	Aggregation	Compatibility	CFPM	MFPM-1	MFPM2
left	0.000	0.001	0.051	[0.000, 0.064]	[0.000, 0.494]	[0.000, 0.718]
right	0.828	0.634	0.550	[0.341, 0.812]	[0.506, 1.000]	[0.282, 1.000]
below	0.000	0.027	0.166	[0.000, 0.282]	[0.008, 0.682]	[0.000, 0.643]
above	0.172	0.338	0.500	[0.188, 0.658]	[0.318, 0.992]	[0.357, 1.000]

Table 2: Results obtained for the object B of Fig. 1 with respect to reference object R.

Concerning the comparison between the existing approaches, several examples have been given in [6] and [5]. Both agree on the limits of centroid methods. In [6], it is noted that the angle histogram depends on the distance between objects: if the objects become farther from each other, the histogram concentrates around one value and the results become closer from those obtained by the centroid method. We should remark that this behaviour is likely to be observed for any method depending on angle computation.

The aggregation method has been proposed only with a weighted mean as aggregation operator. We suggest that this method could be generalized using other aggregation operators, making use of the large variety of fuzzy combination operators. This would allows the user to choose an operator whose behaviour and properties match the requirements related to the application at hand, including subjective ones if necessary [1].

The compatibility fuzzy pattern matching method has a good interpretation as stated above. It is computationally less expensive than the compatibility method but still requires the angle histogram computation, which is the most important part in the computation cost. Also due to the angle histogram, it will be difficult to generalize to 3D. Indeed, two angles are necessary in 3D to define the position of a line with respect to coordinate axes, and this will make the process much more complicated. One important advantage of this method is that it provides two evaluation numbers (or equivalently an interval). This will be detailed below, since this property is shared with the MFPM method.

The morphological fuzzy pattern matching approach has several advantages to our point of view:

- It makes use of spatial fuzzy sets, directly related to the objects under study.

- It takes into account the morphology of the objects: the comparison is not made on a reduced information like a point (dimension 0), nor on a derived function (dimension 1) like angle histogram (which may lead to counter-intuitive results in case of different shapes with same angle histogram), but use the information directly in the image space (2D or 3D).

- It allows for an easier adaptation to any direction of interest. For instance the two ways suggested to compute $\mu_{rel}(R)$ are valid for any direction. On the contrary, the approaches of Section 2 necessitate to define explicitely a fuzzy function for any direction of interest.

- It allows for a simple generalization to 3D, in particular if $\mu_{rel}(R)$ is defined according to the second method: the band around the objects will be delimited by two parallel planes, which makes the computation of an angle between a line and one of these planes easy. The other steps of the method remain exactly the same as in 2D.

- Like for the CFPM approach, it provides 2 values for the evaluation of a relation, one of them being pessimistic and the other optimistic. Again, this leads to a great flexibility in the evaluation, which can be made more or less severe, depending on the application. This can be particularly useful in pattern recognition when relative spatial position is used along with other criteria. The degree of severity can then be chosen according to the importance of the criterion with respect to the other ones, or according to the satisfaction values of the other criteria. The interpretation of the obtained values in terms of possibility and necessity can then be exploited in the framework of possibilistic multi-criteria aggregation, as well as in the context of Dempster-Shafer evidence theory.

Unlike the MFPM approach, all other approaches satisfy some kind of symmetry, in the sense that the following property holds: $\mu^R_{right}(A) = \mu^A_{left}(R)$. Concerning "reflexivity" (in the sense of relationship of an object with itself), we can observe various behaviours with angle histogram based methods. For instance for object R of Fig. 1, the angle histogram with respect to itself is almost flat. On the contrary, for object B of this figure, the angle histogram is not flat at all. This means that, in the sense of these methods, an object may have privileged relative positions with respect to itself. This can be questionable. However, this problem does not occur with the MFPM method if the object is included in the area which has membership value 1 in $\mu_{rel}(R)$ (this is the case for the two proposed approaches). Therefore the MFPM method can be considered as reflexive, in the sense that any object will totally satisfy any relation with itself: $\mu^A_{rel}(A) = 1$.

5. FUZZY RELATIVE POSITION BETWEEN FUZZY SETS

This Section is dedicated to the comparison of the described methods in light of their possible generalization to fuzzy sets, i.e. when both objects are represented by fuzzy sets. In [6] it is suggested to compute a weighted angle histogram, where the weight associated to the angle $\theta(a, b)$ is the minimum (or any other t-norm) of the membership degrees of a and b to the fuzzy sets they belong. In [5], an approach similar to the one originally proposed in [3] is used: the fuzzy relation is evaluated between the α-cuts of both objects and then integrated over all values of α. This approach has also been used in [2] for one definition of fuzzy mathematical morphology. However, this can be computationally expensive, depending on the precision of the quantization of membership functions. The weighting approach of [6] is simpler in this sense. As for the two approaches proposed in this paper, following remarks can be made: for CFPM approach, the fuzzy aspects of the sets has to be taken into account during the angle histogram computation step. Therefore, the same methods as the ones described in [6] and [5] apply. For the MFPM approach, as already mentioned, it applies directly, without any modification, to fuzzy objects. This is an additional advantage of this approach.

6. CONCLUSION

We proposed in this paper two new approaches for defining relative position between objects in images, based on a fuzzy pattern matching approach. In particular the second one presents several advantages over existing definitions: it is flexible, it takes morphological information about the shapes into account, it is consistent with intuitive understanding, it is directly appliable for 3D and fuzzy objects, and it provides an evaluation as two values or equivalently as an interval, which can be useful for further purposes (e.g. combination with other criteria). Since relative spatial position is an essential criterion for recognition of complex scenes, we intend to use the methods proposed in this paper in a fuzzy structural recognition framework, in combination with other criteria like distance, adjacency, inclusion, and node attributes. This is the scope of future work.

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